A duality in recursion to analyse digital society

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Abstract

The concept of Recursion occurs in several contexts: mathematical reasoning (Poincaré), computability theory (Turing, Church), cybernetics (von Foerster), linguistics (Chomsky), epistemology (Cassirer), sociology (Luhmann) and others. It is remarkable that recursion is also emerging as a fundamental key in unraveling biological processes (Maturana, Varela, Damasio, Edelman). We show that there is an isomorphism between biological recursion and all other type of recursions and it is precisely this isomorphism which gives order and therefore meaning to the multiplicity of our experience.

A duality in recursion to analyse digital society

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Abstract

The theory of numbers, the theory of computation and well known biological and neurological studies on cognition and consciousness all indicate the concept of recursion as a common denominator. Mathematical recursion owes its meaning and properties to a dual relation between its results, which always constitute a sequence, and the operator that generated them, which is instead invariant. We propose that this duality in recursion originates from the duality between the biological homeostatic equilibrium in living systems and the adaptive physico-chemical changes required to sustain such equilibria. Such duality gives order and meaning to the experiences of a living system. One of the many implications of this innovative perspective is that the detachment of computational results from our intuitive order relations can cause a rarefaction of the meaning of data and of its capacity to convey communication. This conclusion can have consequences on the analysis of digital society.

Introduction

The theoretical hypothesis presented in this article has its roots in the role played by the concept of recursion, both in the theory of computation and the main theories of neurobiology of cognition and consciousness. One of the main results of the theory of computation was to establish the equivalence between intuitive notions - that is, not formally definable – such as "effective computability", "algorithm", "mechanical procedure", etc. (Mendelson, 1964: 227-228). The above result was obtained in different ways, all consisting in the reduction of such notions into formally defined abstract computational devices such as the Turing machine, lambda-calculus and recursive functions. However, it has not always been sufficiently highlighted in literature that among these abstract devices, the concept of recursive function[1] plays the dual role of mediator and reference (Odifreddi, 1989: 100-102). As we shall see in the first section, this suggests that such a concept should be thought of as the common denominator of all processes governed by formal rules, whether such processes regard numbers, logical operations, mathematical models describing phenomena, or algorithmic performances that reproduce or even constitute phenomena.

Similarly, theories that present a biological approach in the search for the foundations of cognition and consciousness converge towards the establishment of recursion as a fundamental characteristic of life. The centrality of recursion in biological processes, discussed in the second section, was first proposed by Humberto Maturana (Maturana and Varela, 1980: 5). The most accredited works in the field today rarely use the term "recursion" and rarely refer to the work of Maturana. Nevertheless, this common element is evident (Hayles, 2014: 207). The idea of "reentry" in Edelman and Tononi (2000: 48) is central and defined as a recursive mechanism: "Reentry [...] is the ongoing, recursive interchange of parallel signals between the reciprocally connected areas of the brain". Moreover, Antonio Damasio (1999; 1994), in his study of consciousness and emotions, places at the centre of life and cognition a recursive self-regulation (homeostasis) of living systems which preserves the invariant characteristics of the organism, even though that organism is subject to continuous changes. As we shall see in the third section of this essay, the relationship indicated by Damasio (1999: 133-167) between invariance and change is similar to that indicated by Maturana (Maturana and Varela, 1980: 5) between circular processes and historical processes. We show that the close connection between circularity and historicity of biological phenomena are two faces of the same coin, a duality that define recursion in living systems. We conclude this section by introducing the idea that these two indissoluble faces of biological recursion are represented in mathematics as the equally indissoluble concepts of operator and result of the operation.

In the fourth and fifth section, we analyse the relationship between digital society and recursion. For this purpose, we emphasize that psychological and historical studies on the representation of numerosity lead us to believe that the mathematical distinction between ordinal and cardinal numbers originates at the preverbal level. We point out that this distinction is isomorphic to the one between invariance and variation in biological systems and between operator and its results in

[1] For an introduction to the concept of Recursive Function the reader may refer to the entry "Recursive Functions", of *The Stanford Encyclopedia of Philosophy* <plato.stanford.edu/archives/win2016/entries/recursive-functions/>.

recursive functions. The connection between the fields of biology, the theory of recursive functions, psychology and history of numerical representations allows us to use the theory of Damasio on the role of emotion in rational knowledge to investigate the "feeling" (in the sense of Damasio, 1994: 127-164) of the logical evidence - hence the meaning - that can be attributed to numbers.

The meaning of numbers is found in the operations from which they are generated and that connect them to each other. Without these operations, their meaning becomes rarefied and numbers are transformed into numerals, simple symbols without the logical connection that binds numbers to each other. This problem does not only concern numbers, but all "effective calculable" results and algorithms in general, whether they include numbers or non-numerical symbols or images. In a broader perspective, it affects the digitalization of our interaction with the physical and social world.

1. The centrality of recursion in computation

To understand what recursive functions defined on natural numbers really are, it is good to dwell, albeit briefly and informally, on the concept of number. Careful analysis of the concept of number was first undertaken by Grassmann and Pierce and later investigated and refined by Dedekind, Peano and Cantor (Odifreddi, 1989: 17-122; Adams, 2011: 1-17; Corry, 2015: 223-280). For our purpose, it is sufficient to note that natural numbers are based on the successor function. Indeed, by recursively applying the successor operator to the result it previously generated, an inductive sequence is obtained that is the basis for the concept of natural numbers. In other words, we take 0 (zero) as our first element. Through the recursive application of successor function *S*, *S(0)* becomes number 1, *S(S(0))* number 2, *S(S(S(0)))* number 3 and so on (Odifreddi, 1989: 19).

This structure is confirmed and reinforced in the definition of ordinal numbers, which can be considered as the generalization of natural numbers. Ordinal numbers are constructed through set theory; they are ordered in the sense that if we pick two elements, we know which comes 'before' and which 'after', thus allowing a list of objects to be ordered. John von Neumann (in van Heijenoort, 1967: 336) gave the following definition of ordinal numbers: "Every ordinal is the set of the ordinals that precedes it." To understand this definition, we look at how the set sequence called "ordinal numbers" is constructed. The rule of sequence construction is that every new set generated must contain all those generated beforehand. The initial term of the sequence is the empty set, which we indicate by ∅. Thus, at step 0 we have only ∅; at step 1, set {∅} is generated, which contains the element produced by the previous step; at step 2, set {∅, {∅}} is generated, which includes elements produced at steps 0 and 1; similarly, at step 3 we have {∅, {∅}, {∅, {∅}}} or all the elements produced at steps 0, 1 and 2; and so on. Formally, this process is generated by the recursive application of the successor operator as defined in set theory[2]. Therefore, both natural and ordinal numbers are reducible to the successor operator.

If ordinal numbers give the position of an element in an ordered sequence, cardinal numbers express its numerosity. We can formally define the latter starting from ordinal numbers: the cardinality of set *X* is the smallest ordinal number equinumerous to *X*. For example, if set *X* has a cardinality of 3, it is tantamount to saying that its elements can be matched one by one with the elements of set 3 = {∅, {∅}, {∅, {∅}}}.

From these considerations on natural, ordinal and cardinal numbers, we can establish a first point which is of great importance for the arguments developed in this essay: the concept of number requires the concept of successor as its very foundation. A second point is that the successor plays a crucial role in the construction of recursive functions in general. There are three recursive functions called "initial" that constitute the three 'bricks' of which any recursive function is constructed: "zero", "projection" and "successor". While the first two have the task of initializing the recursive process, the successor generates the sequence in a similar way to what we have seen with numbers. Even the rules by which the simpler recursive functions are used to compose more complex ones preserve the logic of the successor function[3] (Moore and Mertens, 2011: 240-249). This makes recursive functions effectively calculable by executing a finite and sequential number of steps. Hence, they represent one of the possible formalizations of the intuitive notion of algorithm (Sipser, 2006: 154).

In addition to the use of recursive functions, the theory of computation shows that algorithms are formalizable in other ways (Odifreddi, 1989). One of the most interesting is the Turing machine

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[2] The formal successor definition of set *X* is: *X'* = *X*∪{*X*}, where *X*' stands for the successor of *X* and U is the union operator. Thus, when *X* is ∅, *X*' will be {∅}; when *X* is {∅}, *X*' will be {∅, {∅}} and so on (van Heijenoort, 1967: 346).

[3] These are the rules of substitution, recursion and the μ-operator. Here are some examples: If the successor function is S(n) = n + 1, the exponential function f(n) = b^n can be calculated by recursion writing f(0)=1 and f(S(n))=bf(n). The first terms of the recursion are f(0)=1, f(1)=b, f(2)=b^2 and so on. Arithmetic operations are also recursive functions. For the sum we have add(m,0) = m and add(m,S (n)) = S(add (m, n)) such that the first terms of the recursion with m = 1 are add(1,1)=S(add(1,0))=S (1)=2, add(1,2)=S (add (1,1))= (2)=3 and so on. For simplicity and brevity, we do not consider an example with the μ-operator.

(Turing, 1936; Moore and Mertens, 2011: chapter 7), an abstract computing device whose purpose is to explore the extent and limits of what is computable. Turing also showed that his machine is not only able to perform manipulation of symbols but can also simulate any other machine. In other words, it is possible to define a universal machine. The Turing universal machine implies the concept of a programmable machine such as modern computers. Yet what class of problems can be solved with Turing machines? Where is their computational horizon located? These questions are answered by two theorems that demonstrate the complete equivalence between Turing machines and recursiveness: "Every recursive function is Turing machine computable" (Odifreddi, 1989: 54) and "Every Turing Machine computable function is recursive" (Odifreddi, 1989: 99). In other words, any function that can be constructed with a recursive process can be programmed and computed on a Turing machine and vice versa. That is, the set of recursive functions constitutes the boundary in which any Turing machine is placed.

The same equivalence has also been demonstrated for every other formal device (algorithm formalizations) capable of computing any effectively calculable function (lambda-calculation, Markov algorithms, flowcharts, calculability according to Herbrand-Gödel etc.) (Odifreddi, 1989: 100). Therefore, all algorithm formalizations are equivalent among themselves, in the sense that what is computable within one formalization is computable within any other. But what makes us think of the recursive function as an ultimate reference, a canonical form of calculability, is the fact that for every pair of formalizations there is a recursive function that can translate one formalization into the other. For instance, in λ-calculation and the Turing machine there is a recursive function which translates the code of a recursive function relative to a Turing machine into the code of the same recursive function relative to λ-calculation (Odifreddi, 1989: 101). We therefore conclude that recursive functions not only constitute the general reference of all other algorithm notion formalizations, determining their computational expressiveness, but that they are also the tools that allow the transition from one formalization to another. The fact that equivalences persist despite the great variability in the details of the various formalizations made Gödel say that we are in the presence of 'a kind of miracle' (Odifreddi, 1989: 102) and Emil Post (in Davis, 1965: 336) conclude one of his articles with these words:

> Indeed, if general recursive functions are the formal equivalent of effective calculability, its formulation may play a role in the history of combinatory

> mathematics second only to that of the formulation of the concept of natural number.

The importance and centrality of recursive functions led Alonso Church to formulate the thesis that *Every effectively computable function is recursive* (in Davis, 1965: 100; Odifreddi, 1989: 101-123). If the thesis were true, as many believe, any algorithm the human mind can conceive, through any device, abstract or concrete, symbolic or mechanical, would always be equivalent to a recursive function; it would have the same computational expressivity. The concept of recursion or recursive function would have the same unifying role that Newton's mechanics had for terrestrial and celestial phenomena or Maxwell's equations for electricity and magnetism.

2. Recursion in the biology of cognition: Maturana

In the introduction to *Autopoiesis and Cognition*, Maturana (Maturana and Varela, 1980: XIX-XX) distinguishes the "organization" from the "structure" of a living system: while the organization is circular and invariant, the structure continually and irreversibly changes, precisely to ensure the stability of the organization despite “perturbations” (Maturana and Varela, 1998) from the environment. The structure is made of material components which interact according to the relations specified by the organization. If material interactions within the structure do not take place according to the organization, the living system dies. It is the organization that qualifies the system and identifies it as a unit belonging to a certain type of living system (Maturana and Varela, 1980: XIX-XX).

Organization is definitely "circular" and “constitutes a homeostatic system whose function is to produce and maintain the same circular organization, determining that the *components* that specify it be those whose synthesis or maintenance it ensures” (Maturana and Varela, 1980: 9). Hence, the living system does not directly experience the environment, but its own reactions to environment perturbations which in turn maintain its equilibrium (Maturana and Varela, 1980: 9-11; 1998). This "operational closure" (Maturana and Varela, 1998: 89, 163-166) of living systems led Maturana (Maturana and Varela, 1980: 8) to state that "everything that is said is said by an observer". Here, he implies that when a living system observes another living system or its own, it maintains and synthesizes the components of its organization, but not the components of the observed system. Hence, the experience is always and only given as a maintenance of the observer's organization (Maturana and Varela, 1980: 9-12, 26-29).

It is therefore in the organization that one must look for something invariant in cognition (Maturana and Varela, 1980: XX). Yet it is impossible to directly experience the organization without the structure. The organization exists only in terms of the material interactions of the structure (Maturana and Varela, 1980: 88-95). But the components of the structure are continuously regenerated so that new ones are materially different from old ones, yet functionally invariant. Thus, the material structure varies sequentially and not circularly as does the organization. Maturana (Maturana and Varela 1980: 26-27) specifies that the structure changes in neither a circular nor arbitrary fashion, but inductively. If the organization dynamic is circular, its materialization in the structure is "historical". A living system develops (grows) and degenerates (ages) irreversibly even though its organization remains the same. This historicity is specific to recursion. A recursive process produces a sequence in which each term is the result of an identical operation that is reapplied circularly to the result of the previous step in the sequence:

> Operationally, a recursion occurs only as a historical phenomenon because it is only in reference to a succession of events that the repetition of an operation is a recursion. That is, a recursion is the repetition of a circular process that an observer sees coupled to a historical phenomenon in a manner that he or she can claim that, in the historical flow of that phenomenon, that repetition results in the reapplication of that process on the consequences of its previous occurrences[4].

As we will see in the next section, it is the constant reference to the organization that gives an order to multiplicity. The tendency we have to reduce experiences to a coherent unit, to seek regularity in them, would appear to be due to this reference.

The dual nature of living systems as invariant and historical is also found in the recursive functions of formal logic. As underlined in the first section, a recursive function, as an operation, is invariant, but in terms of results it constitutes an inductively produced historical sequence. Therefore, the results of a recursive function also owe their unity and meaning to the fact that they belong to a

[4] This English quotation is a translation from the following original text in Spanish: Operacionalmente una recursión ocurre sólo como un fenómeno histórico, porque es sólo en referencia a una sucesión de eventos que la repetición de una operación es una recursión. Esto es, una recursión es la repetición de un proceso circular que un observador ve acoplado a un fenómeno histórico de manera tal que él o ella puede sostener que en el flujo histórico de ese fenómeno, esa repetición resulta en la replicación de ese proceso a las consecuencias de sus ocurrencias previas. (Maturana, 1997: 67).

sequence generated by the invariant operator[5]. Without reference to the "successor", numbers are transformed into simple numerals, that is, symbols deprived of mathematical meaning. It is the operator that generates the order relations that transform simple symbols into numbers.

The difference between the mathematical function as an operator and as a sequence of its results could clarify how it is possible to go from a multiplicity of data to a unitary concept. From his neo-Kantian point of view, Cassirer explains this possibility in a way analogous to ours. As long as we remain limited to individual perceptions, for example the position of a body in space, we can never experience movement as a unitary phenomenon. Kepler would have never found the geometric rule that underlies the orbit of Mars if he had just considered the positions of the planet as a simple set of points. The crucial step for him was to observe those positions with the ellipsis concept in mind. Only then did they lose the condition of experiences without a unitary sense, acquiring that of "laws" which regulate the continuous motion of the planets:

> The individual position of Mars, which Kepler took as a basis, following the observations of Tycho Brahe, do not in themselves contain the thought of the orbit of Mars; and all heaping up of particular positions could not lead to this thought, if there were not active from the beginning ideal presuppositions through which the gaps of actual perception are supplemented. What sensation offers is and remains a plurality of luminous points in the heavens; it is only the pure mathematical concept of the ellipse, which has to have been previously conceived, which transforms this discrete aggregate into a continuous system (Cassirer, 1923: 118-119).

Based on the work of Maturana, we believe we can legitimately deduce that the need and possibility of reducing a multiplicity to a unitary concept, thus giving meaning to the multiplicity, has its origin in the relationship between organization and structure. The recent works of neuroscience in the field of consciousness and cognition seems to confirm this hypothesis.

3. Recursion in the neurobiology of consciousness: Damasio and Edelman

[5] Also the concept of "eigenvalue", conceived by Von Foerster (1976) and discussed by Kauffman (2003), justifies this statement.

Neuroscientist Antonio Damasio (1999; 1994) found a close link between emotion and rationality. The theoretical framework in which he places this association is given by the idea that the reactions of an organism to environmental changes are driven by the need to preserve homeostatic equilibria (Damasio, 1999: 133-145; 1994: 135). Such reactions are basically emotions which arise involuntarily to induce the body to maintain homeostatic equilibrium in spite of environmental changes. Both in the phylogenetic and ontogenetic sense, these reactions are initially unconscious. Only at a later stage are they felt ("feeling" of emotion), giving rise to the emergence of a "nuclear" consciousness, which represents a situational consciousness not yet able to go beyond the here and now. Simultaneously with this point in evolution, mental life emerges in terms of "images". Images arise in connection with the feelings of emotions and therefore have implicit positive or negative categorizations according to whether the corresponding emotions constitute responses to favourable or unfavourable changes to the maintenance of vital conditions (Damasio, 1994: 114-164; 1999: 168-194). Since images are the basis of any rational process, reason is always immersed in feelings and emotions and therefore in the mechanisms of biological regulation (Damasio, particularly 1994: 78-79, 115).

Thus, images act as "somatic markers" (Damasio, 1994: 165-201) in all rational processes which are always decision-making processes. They steer us towards some options and not others, thereby representing the source of both the rationality of practical action (personal and social domain) and formal or abstract thought. Somatic markers act in an obvious way in practical rationality, which more directly concerns decisions related to survival. However, in abstract rationality "they would still act covertly to highlight, in the form of an attentional mechanism, certain components over others", leading one to undertake some reasoning or to logically connect some elements rather than others (Damasio, 1994: 189-191).

The association of emotion with the learning processes of both social and abstract intelligence has been empirically verified in many cases by Damasio (for instance, 1994: 205-222). Although patients with lesions to the frontal lobes do not appear to be affected by the impairment, they are incapable of producing generalizations through induction and to deduce conclusions not previously learned. They are in the condition of "knowing but not feeling", that is, they know how to behave and how to reason in situations experienced prior to brain damage, but are not able to associate feelings and emotions with the decisions taken and this compromises their ability to learn. Subjects who cannot feel emotion due to brain impairments only differ from "normal" individuals when they have to face new situations and problems (Damasio, 1994: 217-222).

The cited studies of Damasio suggest that without feeling emotions it is difficult to see a stable rule in a multiplicity of facts. Recalling Cassirer's example of the orbit of Mars, we can hypothesize that Kepler had to use emotional sensations - which are probably the basis of creativity - to unify the positions of Mars under the concept of elliptical trajectory. Thanks to the feeling of emotion, something emerges that leads us beyond single experiences, not simply associating preassigned terms, but creating association rules that redefine them.

The theories of Maturana and Damasio converge on central points: the stability of functional equilibria within the organism. If in Damasio this component of stability is the concept of homeostasis, in Maturana it is the concept of organization circularity. Neither concept is identified with fixed material elements, but with fixed relationships: stability is found in relationships and not in the terms through which they connect. On the contrary, terms are subject to constant modifications that are allowed provided they foster the stability of vital conditions.

With regard to cognition, for Maturana the historical sequences in the structure to maintain homeostatic equilibrium represent the cognitive development of the living system. In general, for a living system the experience of the environment is a recursive accumulation in which every explanation "is the reformulation of an experience in terms of another experience" (Maturana, 2001: 28-29). This is because an observed experience is a state of the organism’s cognitive domain which is distinguished when a new state of the cognitive domain of the organism emerges (Maturana and Varela, 1980: 26-30). Therefore, all cognition develops recursively through distinctions of changes generated to maintain the stability of the organization in response to perturbations caused by the environment (Maturana, 1995). Similarly, in Damasio cognition is a historical development of changes that ultimately achieve the maintenance of functional equilibria at all three levels which he distinguished: at the unconscious level of the proto-self, at the situational level of "nuclear" consciousness and at the "autobiographical" level of extended consciousness (Damasio, 1999: 172-188; 192-199; 1994: 160-164).

From the physiological viewpoint (neural and chemical-biological) as well as the phenomenological (mental representations), cognition proceeds with a recursive trend to preserve a biological equilibrium. Through the generation of historical sequences, the stability necessary for life is reproduced in ever-changing forms. Hence both continuous changes and constant reference to some invariant represent the biological necessities of cognition. Edelman and Tononi (2000) also converge on this point, which is conceptually, though not terminologically, common in both Maturana and Damasio.

Edelman and Tononi developed a neurological theory of knowledge based on the concepts of selection and recursion. They place their theory inside Darwin's with regard to phylogeny, but the novelty lies in the proposal of a selection mechanism which is also active on the ontogenetic level. Although the anatomical formation of the brain is undoubtedly bound to genetic inheritance, from the embryonic stages an immense repertoire of neural circuits is created whose persistence depends on whether the activation of the neurons is more or less synchronous: "Neurons that fire together, wire together" (Edelman and Tononi, 2000: 83). As a result, some groups of neurons have more connectivity than others. Superimposed upon this first stage called "developmental selection", another stage called "experiential selection" lasts throughout the life of the organism, in which synaptic bonds within and between neural groups are selected on the basis of behavioural experiences. These two mechanisms lead to a differentiated and specialized manifold of neural links (Edelman and Tononi, 2000: 79-84).

The different and specialized neuronal groups are dynamically integrated thanks to "reentry" (Edelman and Tononi, 2000: 85). Reentry "is the ongoing, recursive interchange of parallel signals between reciprocally connected areas of the brain, an interchange that continually coordinates the activities of these areas' maps to each other in space and time" (Edelman and Tononi, 2000: 48). The spatio-temporal correlation recursively generated by reentry lies at the basis of the phenomenological integration of an experience. Based on tests conducted with digital models, the authors (2000: 113-120) formulate the hypothesis that in the interval of a few hundred milliseconds, all the neural groups involved in the perception of a given scene are active in a synchronized manner. However, in the first tenth of a millisecond only neuronal groups excited by the attributes of the same object are synchronously activated, while those related to different objects act asynchronously. Reentry therefore acts both at the level of "categorical perception", which is unconscious and whose recursion takes a few tens of a millisecond to integrate each perception of an object, and at the level of the widest integration of a scene, which is conscious, overlapping the first and whose recursion must last longer to achieve the whole scene integration (Edelman and Tononi, 2000: 69, 118).

The integration of brain activities from which consciousness emerges is not coordinated by any particular area of the brain, but by a "dynamic nucleus" that is continuously reconstituted through the synchronization of different areas. It is both the result and the cause of reentry (Edelman and Tononi, 2000: 49, 139-154). We can therefore say that the spatio-temporal correlation between brain maps acts as the 'rule' that guides the recursion represented by reentry. For each cycle of recursion, the correlations generated between the maps are always new, but the necessity for correlation is constant. Reentry has the stable function of generating synchronization and coordination between maps. It is a biological necessity with circular dynamics that generates an historical sequence. The subsequent forms in which integration occurs between maps constitute such a sequence, while the continuous reconstitution of spatio-temporal correlations between them constitutes the recursion operator. At the level of mental phenomenology, the general result of neurological reentry is the historical sequence of scenes and / or behaviours generated as subsequent integrations of experience data. Scenes and behaviours are always different, but the need to synchronize and coordinate them is constant.

In the next section we analyse in detail the relationship between the indissoluble biological link of invariance-historicity and the mathematical link, equally indissoluble, of operator-numbers. This analysis considers historical and psychological research on the origin of numbers and arithmetic calculus. We also present some consequences that can be drawn from our analysis for computer-mediated communication.

4. Invariance and change in numbers

The mathematical definition for the concept of function presents a problem: as an abstract concept it fails to express the intrinsic generative character of a function. To better understand the point, one must consider this definition: a function is a univocal relation[6]. Now, a relation is any subset of a Cartesian product. A Cartesian product between sets A and B is the set of all ordered pairs that can be formed by associating each element of A with an element of B. Thus, without prejudice to the condition of univocity (i.e. each element of A corresponds to one and only one element of B), a

[6] More exactly: a function f is a binary relation such that if $<x, y> \in f$ and $<x, z> \in f$ then $y = z$.

function from A to B is any subset of all these pairs. This definition does not entail a generation from A to B, but any pairing between pre-established terms which are the elements present in A and B.

This definition creates a gap between the general concept of function and a concretely assigned function[7]. Consider the assigned function $y = x + 3$. It tells us that if x is 2, then y is 5. If we get this result according to the general concept of function, we must know in advance that the pairs (1, 4), (2, 5), (3, 6), (4, 7), and so on, belong to the function. Therefore, if x is 2, y is 5 because the pair (2, 5) belongs to the function. In reality, this result is not obtained in this way, but by generating 5 starting from 2 through the operation "add 3 to the given number", where this operation is ultimately based on the successor (see note 3).

From Galileo (1980: 374-375) onwards, modern scientific thought has avoided asking questions about the "essence" of things to avoid falling back into the tendency towards substantialism of Aristotelian philosophy (Cassirer: 1999). This is probably why researchers have been reluctant to investigate the origin of the concept of function, which in essence seems to be generative. It is never a pairing between given terms, but a rule that generates one term from another. In the case of the function $y = x + 3$, it is evident that the rule is to add 3 to the value assigned to x. But if we ask what the sum is, we inevitably end up explaining it with the numerical succession and then with the successor operator. Still, the backward process is blocked, since it is impossible to define what the successor operator is, regardless of the numbers it generates. The problem is identical for any other effectively computable function. To continue the backwards search process for investigating where the origin of the successor operator is located, it is necessary to go beyond the disciplinary field of mathematics. In our hypothesis, the successor operator originally relies on the recursiveness of our biological constitution.

Gallistel and Gelman (1992) argue that, even at the preverbal level, the generative operators of numbers underlie the experience of numerosity. They report psychological experiments made on animals and human beings which suggest that quantification is anterior to language and is the result of non-verbal arithmetic operations. They call such preverbal mental entities "numerons". To better explain this idea, they first highlight the distinction between the "estimator" and "operator"

[7] This problem was also noted by Bertrand Russell (2009: 265-270) in the context of propositional functions.

processes. The former "determines the mapping or reference relations between the numerons and the numerosity to which they refer". The latter "processes one numeron (unary operators) or two numerons (binary operators) to produce another numeron". In the estimator process, a numeron is a "category" and "refers to all sets of given numerosity". By contrast, in the operator process, numeron is a "concept" and "plays a unique role in a system of mental operations isomorphic to at least some arithmetic operation" (Gallistel and Gelman, 1992: 44).

The reader can see that the distinction between the "estimator" and "operator" preverbal processes corresponds to the distinction we highlighted above between the terms and operator of a function. In one case the numeron is a predetermined category that represents all the sets of given numerosity; in the other, it is a concept that represents the activity of a generative operator that produces a new term based on previous terms.

The experiments reported by the authors first confirm the existence of entities that represent the preverbal number or what they call the numeron. Secondly, they suggest that the operator process also underlies the estimator process, generating the numeron as a concept, which then represents the reference for the numeron as a category (Gallistel and Gelman, 1992: 47-55). In other words, it seems that even at the preverbal level, the cognition of a quantity must be generated in an ordinal manner and then used in a cardinal way. For the authors the operator process corresponds to an original neurobiological activity. It would be a result of evolution because of the survival advantages it offers (Gallistel and Gelman, 1992: 46). To use Maturana's terminology, it would be one of the results of the phylogenetic drift that led to the human species.

The psychological theories proposed as alternatives to that of Gallistel and Gelman favour a "cardinal vision" of the number, as opposed to "ordinal". To use Gallistel and Gelman's words, they are more oriented towards numeron as a category than as a concept. For example, Stanislas Dehaene (2011: 58) argues that preverbal numerosity is not the result of a generative operator but of direct perception. Certain neurons would specialize in the perception of each concrete numerosity: for example, the numerosity of six items would make a given neuron reach the critical threshold of activity, causing it to produce a discharge that represents the sensation of the numerosity of six (Dehaene, 2011: 17-23).

In our opinion, the theoretical hypothesis of Dehaene, like all those oriented towards the cardinality of the number, is hard to reconcile with the view of living as a complex system. The idea that numbers are predetermined entities of external reality, recognized by the nervous system through the specialization of neurons, cannot be reconciled with the concept of cognition based on autopoiesis (Maturana), on self-adaptation of the body (Damasio) or on reentry (Edelman).

If, as we believe, only ordinal numbers are directly linked to the recursive nature of our biological structure, this could have various consequences for the analysis of digital society. One of these concerns the effectiveness of computer-mediated communication. The basic functions of electronic computation operate according to the logic of cardinal numbers. In the definition of function formally adopted by mathematics, electronic operators are defined by pairing predefined terms without using a generative procedure. Improperly but effectively simplified, our biology is 'ordinally oriented', while electronic computation is 'cardinally oriented'. Another way to express the same concept is given by the distinction between syntax and semantics in logic. Syntax is a set of rules for manipulating an assigned set of symbols. Semantics is the meaning a string acquires once it has been assigned an "interpretation" by the symbols (Hunter, 1992: 9). While our biology is based on semantic computation, governed by emotions, feelings and the intuitive meaning resulting from them, electronic devices are based on syntactic computation, obeying pre-established and formally defined rules (truth tables of binary logical operators) independent of the meaning that can be attributed to symbols or their manipulation.

Let us clarify this last argument. An electronic computer is a network of 'logical blocks' constructed to correspond to propositional calculus. Suppose one wants to digitalize the statement 'A implies B'. Assigning a value of 1 to statements A and B and a value of 0 to their negations, the Boolean logic block of the implication is constructed to only admit the pairs of values <1, 1 >, <0, 1>, <0, 0>, excluding <1, 0> (see Table 1). For example, statement A: 'I put my hand in the fire', and B: 'I burn my hand'. The logical block that digitally represents the implication only admits 1 = 'I put my hand' and 1 = 'I burn', 0 = 'I do not put my hand' and 1 = 'I burn' (for a different reason from the contact with fire), 0 = 'I do not put my hand ' and 0 = 'I do not burn '. Although its results are isomorphic to human behaviour, the Boolean implication expresses a mechanism that differs from the biological one. Indeed, if 'I do not put my hand in the fire' it is not because in this way I can or cannot burn myself (cases III and IV of the table 1). I do not do it simply because, from previous experience, my body reacts in an emotionally negative way to the idea of putting my hand in the fire. Emotion provides immediate evidence that I should not behave in a certain way. As we have seen, Damasio's theory is based on this point[8].

Cases	A	B	If implication is TRUE, the case is
I	1: 'I put ...'	1: 'I burn…'	Admitted
II	1: 'I put ...'	0 'I do not burn…'	Not Admitted
III	0: 'I do not put ...'	1: 'I burn…(for other reason)'	Admitted
IV	0: 'I do not put ...'	0: 'I do not burn…'	Admitted

Table 1. Truth table of logical implication.

In current technology, automatic information operators work differently from biological ones. Subordination of our lives to computer calculations is not necessarily functional to our biology. Norbert Wiener's (1954) concern about the fact that human beings must retain their autonomy with respect to these systems cannot be dismissed as ideological, as Katherine Hayles (1999) does. Hayles' thesis on Wiener's relationship between human beings and technological products of cybernetics suggests that Wiener had, as an unconscious reference, the typical myth of liberal ideology: an autonomous individual, free in thought and initiative. As a man of his time, Wiener may have experienced this kind of conditioning. However, as our previous analysis indicates, the problem he points out has a real foundation. Hayles (2014: 202-208) cites the theories of Damasio and Edelman as important references and recognizes the centrality in Maturana's thesis on recursion in living systems. They all focus on the idea that human cognition has at its centre our biological homeostatic equilibrium. Therefore, underestimating the need to preserve the human way of knowledge means underestimating the need to preserve human biological mechanisms as they are configured in the current state of our species.

5. Numbers and Communication

[8] Maturana (2001: 46) introduces another important reflection: emotion changes reality. If a beetle is quietly eating a crumb of bread on the floor of my kitchen and I come in and scream and cause the beetle to panic, it will flee and will not be able to eat anymore; yet, objectively, the breadcrumb is still there. Similarly, our perception of a fire will be different depending on the emotion we attach to it.

The order relation expressed through numbers is an integral part of man's adaptation to the communicative environment. To clarify this statement, we present an anecdote reported by Ifrah (1998: 21):

> I once knew someone who heard the bells ring four as he was trying to go to sleep and who counted them out in his head, one, one, one, one. Struck by the absurdity of counting in this way, he sat up and shouted: "The clock has gone mad, it's struck one o'clock four times over!"

The four clock chimings inform us that it is four o'clock only if they are part of a communication code that interprets them as belonging to a single ordinal sequence. Otherwise, they represent only four distinct entities that express a cardinality. A clock is constructed to accurately follow a communication code that acts as an interface between society and the physical environment - in this case, astronomical time. But if its machinery breaks down and begins to beat unexpectedly and irregularly, without the possibility of clearly distinguishing the sequences, it becomes impossible to say that the clock is indicating a specific time. If a machine that has to provide information no longer respects the human communicative code, it loses its meaning and usefulness in interfacing with the environment.

The history of modernity has already indicated that this problem exists. The authors (2014, 2016a, 2016b) of this essay argue that since the late Middle Ages, there has been a progressive penetration of computing logic into practical life, in forms that are in fact algorithmic. This is possible thanks to two different epistemological operations that identified the mediators between the mental world of mathematics and the concrete world of 'reality'. For the physical and economic world, such mediators are the modern concepts of space, time and exchange value. The role of these mediators is to enable the link between reality and the mental sphere by transforming both physical and economical experiences into mathematical dimensions. For industrial and service organizations, the mediators were instead the formal classifications of people and their actions, according to the typical theoretical systematization called Taylorism (Taylor, 1911). As opposed to space, time and value, which are pure computational dimensions, classifications are ontologically predetermined entities. They have been standardized through operational definitions with the aim of replacing communicative interaction with centralized algorithms, thus applying the logic of computation in this field. Yet the transformation of the workforce into standardized units manipulated by bureaucratic algorithms has separated algorithms from communication. This model, as we know, generated unsolvable contradictions and is now obsolete (Blau, 1956; Blau and Scott, 1962; Crozier, 1963; Porter, 1985; Johnston and Lawrence, 1988). However, in areas where calculation operates as a communication tool, no phenomenon of obsolescence has occurred. On the market, for example, individual actions interact directly with economic dynamics. Market coordination is generated spontaneously as an emerging phenomenon starting from those individual actions. Therefore, individual calculations generate a self-organized system that adapts to changes in the same way as living systems (Hayek, 1945; 1952). Unlike bureaucracy, the market does not appear to be in a crisis or at all obsolete, which is due to the fact that, here, computation operates as an instrument of communication.

Algorithmic logic has worked as a mediator, both preserving and impeding communication in the ways indicated above, which are converging. The algorithmic systems that mediate social interaction can nowadays drastically reduce and perhaps eliminate the technocratic classification and make the interface open to autonomous user projects, such as the Linux platform (Lanier, 2010). Instead, we witness the progressive domination of "proprietary" systems - jealously guarded by secrecy - whose purpose is to develop dynamic classifications in which to enclose individuals. The so-called "recommendation algorithms" aim to encapsulate network users in heteronomous interfaces, according to ‘profiles’ that are continually and automatically redefined on the basis of user behaviour. This is the phenomenon that Pariser (2011) called "filter bubbles". In a way similar to bureaucracy, filter bubbles interrupt human-to-human communication interaction and seek the coordination of human behaviour through subordination to centrally managed algorithms designed by a technocracy.

Dependence on algorithmic systems could also threaten the ability of our society to achieve scientific advancement. In his monumental work on the history of numbers, focusing on the relationship between society and the symbolic manipulation of quantities, Georges Ifrah (1998: 20-22) argues that advances in such manipulations always occurred thanks to the concept of order relation. After recalling that numbers have "two complementary aspects: cardinal numbering, which only relies on the principle of mapping, and ordinal numeration, which requires both technique of pairing and the idea of succession", he concludes this quote by stating that arithmetic is based on the concept of ordinality:

> For whereas in practice we are really interested in the cardinal number, this latter is incapable of creating an arithmetic. The operations of arithmetic are based on the tacit assumption that we can always pass from any number to its successor, and this is the essence of the ordinal concept (Ifrah, 1998: 22).

The practical problem of counting a potentially infinite sequence using only a small number of symbols has been satisfactorily achieved thanks to the positional numbering system and the number zero (Ifrah 1998: 346). Both discoveries occurred in different periods and civilizations, but they were generally guided by the order relation expressed by the successor function (Ifrah, 1998: 23-439). The result, which is the positional system that we use today, is based on a double order of recursions: the first refers to the sequence of digits ranging from 0 to 9 and the second to the sequence of positions - from right to left - of these digits in the string that represents the number (units, tenths, hundredths, etc). Without understanding the number as the recursive result of an operation applied to its result, it would not have been possible to reduce infinite quantities to unitary concepts.

Quantitative data are meaningful to us only if ordered and we believe that this order is ultimately due to the recursive trend of the dynamics of our organism. Our experience in general is due to this dynamic, which generates inductive sequences of material changes in the organism to produce our biological balance. Such changes are meaningful, they tie to emotions and feelings only because they represent the way in which the living system keeps the vital equilibria unaltered. We propose that the relationship between the successor function and numbers is a variation of the general relationship between the biological operator (Maturana's organization circularity, Damasio's vital equilibrium and Edelman and Tononi's reentry) and its sequential results. Just as an experience would lose meaning without its relationship with the biological operator, mathematical data lose meaning if they are no longer attributable to order relations, which are ultimately based on the successor function. If we have data presented by algorithmic systems without the possibility of inserting them in order relations, they can lose their meaning.

Final considerations

Recursion is emerging in contemporary culture as a general reference of cognitive processes[9]. It is also indicated as the source of the concepts of number and algorithm. Such findings draw greater strength and significance from the main theories of cognitive neurobiology which focus on recursive processes. We believe we have managed to give a rational basis to the inseparable connection between the biological origin of recursion and its foundational role in numbering and computing.

If one wished to effectively and concisely summarize Maturana and Varela's concept of autopoiesis, Damasio's homeostasis and Edelman and Tononi's reentry, Spinoza's Ethic propositions 6 and 7 (3rd part) could be of great use: "Each thing, insofar as it is in itself, endeavours to persist in its own being." and "The conatus with which each thing endeavours to persist in its own being is nothing but the actual essence of the thing itself." In the context of this essay, such statements[10] must be understood in the sense that biological recursion is not only the process by which a steady state in a living system is maintained, but that it is also the defining feature of life itself. We have shown that a recursive process can be viewed either as a primitive and invariant operator, or as concretely defined by looking at the historical sequences it generates. These modalities are respectively represented as 'organization' and 'structure' by Maturana. The variability and multiplicity of experience is functional to the maintenance of a living system's steady state and, more importantly, the meaning we attribute to such variations and multiplicities originates precisely in the need for survival.

This article suggests that biological recursive processes are isomorphic to recursion in mathematics and, in particular, to the recursive functions found in the theory of computation. Hofstadter's definition of isomorphism, less precise than that used in mathematics, is nevertheless more appropriate in this essay: "The word isomorphism applies when two complex structures can be mapped onto each other, in such a way that to each part of one structure there is a corresponding part in the other structure, where "corresponding" means that the two parts play similar roles in their respective structures". An isomorphism induces meaning. The successor function in number theory, for instance, has the double role of generating natural numbers and, at the same time,

[9] As is widely known, in linguistics recursion has also assumed a central role with Chomsky's (1988) generative grammar theory. Von Humboldt had already recognized that language presupposes "the infinite use of finite means" and it was Chomsky himself who pointed out that this intuition of von Humboldt took on a clear and precise form with the theory of recursion as a finite medium for the generation of the infinite expressions of a language.

[10] The reader interested in exploring the connection between Spinoza's philosophy and the neurobiology of emotions and feelings can refer to the book by Damasio "Looking for Spinoza. Joy, Sorrow and the Feeling Brain".

meaning through the induced order relation. Because of isomorphism, chemical and biological recursion acquire meaning from the "order relation" induced on the products of recursion. The conclusion is that our ability to give order and meaning to the multiplicity of experience is connected to the chemical and biological recursive process indispensable for the maintenance of life[11].

The implications of this conclusion may be numerous. Here, we only stress that a culture which relies on quantitative data disconnected from the underlying order relations can weaken its meaning and capacity to convey communication. This is what could happen when relying on the results of algorithmic systems whose operating logic is unknown.

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[11] "Living systems are cognitive systems, and living as a process is a process of cognition" (Maturana and Varela 1980: 13).

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