Stock market returns predictability: Can we improve it?

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Abstract

This paper explores the possibility of improving the predictability of financial returns via a decomposition method called **"External trend and internal componenets analysis"**. Using some Chinese and American stock market, it approves that the method enhance predictability

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Abstract

Accurately predicting stock market returns can pay off economically not by yielding significant profit, but rather by preventing the loss of a large sum of money. Therefore, stocks with a high degree of predictability gain more attention. This study is intended to investigate the impact of decomposing returns on improving predictability. The decomposition method is used to separate the return local components from the external trend. The approximate entropy technique is applied to quantify their randomness amounts. The results reveal that the decomposition method improved the predictability of returns from S&P500, Nasdaq 100, SSE and SZSE 500 stocks. The outcomes show that using stock absolute value can further enhance its performance. Moreover, this study shows that S&P500 intraday data are more predictable than their daily data. These findings propose incorporating the decomposition method in the prediction process to improve the predictability to maximise the investors profit and minimise their risk.

Keywords: Stock market; Returns; Predictability; Decomposition method; Approximate entropy.

1 Introduction

Stock market management has been gaining importance in the past several years; however, the researcher mainly focused on return forecasting and volatility (Rapach and Zhou, 2013; Pan et al., 2020; Sun and Yu, 2020). In financial economics, the efficient market hypothesis states that future prices can not be predicted based on past prices (Malkiel, 2003). This concept has been continually disapproved in different ways since the 1980s (Lee and Lee, 2009; Rossi and Gunardi, 2018). The dwindling of support among researchers for it was encouraging to explore the stock's returns structure. Moreover, economists lack a fundamental theory behind their complex behavior. By analysing the relationships between the agents using different tools, many studies have been done to extract meaningful information. Researchers generally focused on understanding their correlations for both daily (Forbes and Rigobon, 2002) and intraday time scales (Münnix et al., 2010).

The network theory is a tool used to characterise and classify the different financial instruments' interdependence (Mantegna, 1991). Several methods have been developed to include the non-linearity of stock return's dynamic analysis (Fiedor, 2014c). Most of the analyses use synchronous correlations of equity returns. They have shown a common factor that drives returns, and stocks themselves are arranged in groups (Fiedor, 2014b). Kausik (Chaudhuri, 1997) has found evidence of a single stock market's common trend by an emperical investigation. Therefore, many studies proposed models for its prediction (Yiwen et al., 2000; Wen et al., 2019). Separating the market's global trend from the local effects for stock markets has been a crucial problem. It allows distinguishing whether the stocks are just following the common trend or, on the opposite, they are the

source of their fluctuations.

Analysing stock markets from the point of predictability has attracted many researcher's attention. Scholars focus on testing return predictability for different stock markets (Chen et al., 2010; Lanne, 2002; Bannigidadmath and Narayan, 2016) or examining the robustness of the evidence on stock return predictability (Pesaran and Timmermann, 1995; Campbell and Yogo, 2006; Kostakis et al., 2015). A significant number of these studies used the approximate entropy to analyse financial time series (Darbellay and Wuertz, 2000; Assaf et al., 2021). Because of its suitability for characterising them (Pincus and Kalman, 2004) and its usefulness in quantifying the market's efficiency in stock and foreign exchange(Risso, 2008, 2009; Zunino et al., 2009; Oh et al., 2007). Entropy is a technique that borrowed its concept from mechanics and information theory (Jaynes, 1965; Shannon, 1948), where estimations are affected by the system noise since it requires infinite data series. With simple computations based on the repetitive patterns of time series fluctuations, the approximate entropy method proposed by (Pincus, 1991; Pincus and Huang, 1992) is used to address this problem. To the authors' best knowledge, few publications are available in the literature discussing improving returns predictability. However, studies aimed to enhance the predictability degree are needed to be done.

This paper explores the possibility of improving the predictability of financial returns. Unlike previous studies exploring the relationship between their statistical properties and predictability (Duan and Stanley, 2011; Pan et al., 2005), it uses the separation of the local effects from the global trend imposed by the

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market. The decomposition method based on the independent component analysis approach evaluates the efficiency of local market policies. It offers a better understanding of the system dynamic and hence improves the returns directional prediction correctness. The results show that returns predictability degree after decomposition has been improved. Incorporating the return's absolute value in the process can enhance its performance. Moreover, this study examines the impact of frequency on predictability and find out that high-frequency data are more predictable than daily data.

The remainder of the paper is organised as follows: Section 2 presents data description and the methods and techniques utilised in the empirical analysis. Section 3 discusses the empirical results. Finally, section 4 concludes this work.

2 Data and methodology

2.1 Data

This study is based on 4 stock indices from the USA and China to make the results more convincing. The data has been downloaded from the Wind Financial Terminal platform. Due to data constraints, the dataset for S&P500 (Standard and Poor's 500) and Nasdaq 100 (National Association of Securities Dealers Automated Quotations) extends from January 5, 2010, to May 28, 2019 (389 and 89 stocks), while for SSE Index (Shanghai Stock Exchange) and SZSE 500 composite index (Shenzhen Stock Exchange) covers the period from January 4, 2000, to May 28, 2019 (315 and 245 stocks). This study used 1-minute data to investigate the impact of frequency on predictability for the same stocks listed on S&P500.

1-minuite data cover the period from March 27, 2019 to April 5, 2019 due to consistency and availability of all stocks that will help compare them.

The decomposition method used in this study requires the following data transformation:

$$r_i(t) = \frac{P_i(t) - P_i(t-1)}{P_i(t-1)} * 100, \tag{1}$$

where $P_i(t)$ and $P_i(t-1)$ are the prices at the instants t and t-1, respectively.

2.2 Decomposition method

For a time series of returns $r_i(t)$, $i = 1, ..., S$ and $t = 1, ..., T$, where i refers to a specific stock, the existing methods for the separation of the internal from the external contributions allow writing the time series in the following way:

$$r_i(t) = r_i^{ext}(t) + r_i^{int}(t), \tag{2}$$

where r_i^{ext} represents the impact of the market trend on the stock i and r_i^{int} symbolise the contribution due to purely local factors.

Generally, these methods assume that the local components have a zero average. Under this assumption, Barabasi et al.(de Menezes and Barabási, 2004) have proposed a method to separate the internal dynamics where the following equation can compute the external components:

$$r_i^{ext}(t) = a_i \sum_{i=1}^{S} r_i(t), \tag{3}$$

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where

$$a_i = \frac{\sum_{i=1}^{T} r_i(t)}{\sum_{i=1}^{T} \sum_{i=1}^{S} r_i(t)}, \tag{4}$$

and

$$r_i^{int} = r_i(t) - \left(\frac{\sum_{i=1}^{T} r_i(t)}{\sum_{i=1}^{T} \sum_{i=1}^{S} r_i(t)}\right) \sum_{i=1}^{S} r_i(t). \tag{5}$$

This method can forecast the correct outcome's in specific cases, therefore, Barthelemy et al.(Barthélemy et al., 2010) proposed the ETICA decomposition method that is based on an independent component analysis approach (the external trend and internal components analysis). The context is essentially the Arbitrage Pricing Theory (APT), in which r_i^{int} is the excessive α. The a_i's estimation is not conceptually different from the more established Fama-Macbeth regression techniques widely used for factor extraction. The ETICA methodology is an alternative approach to Fama-Macbeth within the APT context that adds value from a finance perspective. Barabasi et al. (de Menezes and Barabási, 2004) proposed the separation method, where the internal component r_i^{int} has a zero average by definition. Its pricing implications yield the restriction that the elements of the parameter vector α are jointly equal to zero. However, the internal contribution average is expected in many cases to be non-zero; hence this yields incorrect results. The decomposition method assume the independence of the global trend from internal contributions, which are required to be independent of stock to another and the external components so can be written:

$$r_i^{ext}(t) = a_i w(t), \tag{6}$$

where $w(t)$ is the collective trend common to all stocks reacting to it with the

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prefactor a_i, so the authors assumed:

$$r_i(t) = a_i w(t) + r_i^{int}(t). \tag{7}$$

The parameter $\frac{\mu_w}{\sigma_w}$ is estimated under two scenarios (the average of $w(t)$ and its dispersion). The first one assumes that in the absence of the internal contributions:

$$\frac{\mu_w}{\sigma_w} = \frac{1}{S} \sum_i \frac{\langle r_i \rangle}{a_i}, \tag{8}$$

where

$$\langle r_i \rangle = \frac{1}{T} \sum_{t=1}^{T} r_i(t), \tag{9}$$

or by an alternative assumption:

$$\frac{\mu_w}{\sigma_w} = \frac{\langle r^{av} \rangle \bar{A}}{\bar{A}^2}, \tag{10}$$

where $r^{av} = \frac{1}{S} \sum_i r_i$ and $\bar{A} = \frac{1}{S} \sum_i (A_i)$. In this cases μ_w and σ_w can be fixed to: $\mu_w = \langle r^{av} \rangle$ while, $\sigma_w = \langle W r^{av} \rangle$ ($W(t)$ is the global normalized pattern). The second scenario assumes the absence of correlation between A_i's ($A_i = a_i \sigma_w$) and the temporal average of r_i^{int}'s. Barthelemy used the second scenario to estimate $\frac{\mu_w}{\sigma_w}$ since the assumption of the absence of the internal contribution leads to incorrect results (Barthélemy et al., 2010). The parameter $\frac{\mu_w}{\sigma_w}$ is estimated by the slope of an observed linear correlation obtained from the following equation:

$$\langle r_i \rangle = A_i \frac{\mu_w}{\sigma_w} + \langle r_i^{int} \rangle. \tag{11}$$

To consider the case of having a strong correlation (negatively and positively)

we propose the following new approach:

$$corr(A_i, \langle r_i^{int} \rangle) = \pm 1, \tag{12}$$

means by definition that there exist a, and b such as:

$$\langle r_i^{int} \rangle = aA_i + b, \tag{13}$$

by replacing $\langle r_i^{int} \rangle$ in the equation (11), we get:

$$\langle r_i \rangle = (\frac{\mu_w}{\sigma_w} + a)A_i + b. \tag{14}$$

In the absence of any condition about a and $\frac{\mu_w}{\sigma_w}$, we can't separate them from each other [we can get $(\frac{\mu_w}{\sigma_w} + a)$ with a linear regression]. Therefore, to express that the correlation is equal to ± 1, and assume that:

$$A_i = \pm \langle r_i^{int} \rangle, \tag{15}$$

we get then

$$\frac{\mu_w}{\sigma_w} = \frac{1}{S} \sum_i (\frac{\langle r_i \rangle}{A_i} - 1)(corr = 1), \tag{16}$$

and

$$\frac{\mu_w}{\sigma_w} = \frac{1}{S} \sum_i (\frac{\langle r_i \rangle}{A_i} + 1)(corr = -1). \tag{17}$$

After collecting the data, this study applied the ETICA method once its conditions mentioned in Barthélemy et al. (2010) are satisfied. To check the effect of the $\frac{\mu_w}{\sigma_w}$ on the results, it considered the equations (10), (15), and (16) and compared the results obtained for different values. Table 1 summarises the intervals

for the parameter $\frac{\mu_w}{\sigma_w}$.

Table 1: **Intervals for the parameter** $\frac{\mu_w}{\sigma_w}$

/	**S&P 500**	**NASDAQ 100**	**SSE**	**SZSE 500**
corr=1	-0.9369	-0.8638	-0.7886	-1.2919
corr=0	0.0124	0.0473	0.0193	-0.0295
corr=-1	1.0631	1.1362	1.2114	0.7081

Using stocks from S&P 500(daily data), NASDAQ 100(3 days data), SSE(weekly data) and SZSE(monthly data) indices when the correlation between r_i^{int} and A_i is equal to -1, 0 and 1.

This paper extended the external trend and internal components analysis decomposition method algorithm and applied it to $|r_i(t)|$ instead of $r_i(t)$ to enhance the predictability. It is very important and useful since many studies have suggested that signs of returns are predictable (Chronopoulos et al., 2018).

2.3 Approximate entropy

The algorithm of Kolmogorov–Sinai entropy has been shown to work well for real dynamic systems, but even a small amount of noise makes it fail in analysing the system's complexity successfully (Delgado-Bonal and Marshak, 2019). To quantify the concept of changing complexity, *Pincus*, in 1991, developed a new statistic for the experimental data series called "Approximate Entropy" (Pincus, 1991). The study concluded that the application of the K-S entropy was incorrect in some cases, such as the presence of stochastic components.

To solve the K-S entropy limitation, he formulated the approximate entropy ($ApEn$) with the same philosophy. The independence of the $ApEn$ of any model makes it suitable for a different kind of data analysis (Delgado-Bonal and Marshak, 2019). Therefore, it is applicable without any assumption about data. This

is why it is extensively used in different fields. As an input, the $ApEn$ required the pair parameter m, the embedding dimension (a non-negative integer), and the noise filter r (positive real number).

Given a time series $r_i = r_i(1), r_i(2), ..., r_i(T)$ of length T, he defined the blocks:

$$r_i(j) = r_i(j), r_i(j+1), ..., r_i(j+m-1), \qquad (18)$$

and

$$r_i(k) = r_i(k), r_i(k+1), ..., r_i(k+m-1), \qquad (19)$$

the distance between them is:

$$d[r_i(j), r_i(k)] = max_{l=1,2,...,m}(|r_i(j+l-1) - r_i(k+l-1)|), \qquad (20)$$

By letting the value of C_j^m calculating the number of blocks (with length $=m$) similar to a given block, consecutive values be equal to:

$$C_j^m = \frac{d[r_i(j), r_i(k)] \leq r}{T - m + 1}, \qquad (21)$$

The approximate entropy is calculated by:

$$ApEn(m, r, T)(r_i) = \phi^m(r) - \phi^{m+1}(r), \qquad (22)$$

Where

$$\phi^m(r) = \frac{1}{T - m + 1} \sum_i^{T-m+1} log C_i^m(r). \qquad (23)$$

where parameters m and r can be fixed to recommended values. Even the method necessitated data between 10^m and 30^m. It could be applied to data where T=100

(Pincus, 1995; Pincus and Huang, 1992).

According to Pincus (Pincus, 2008), approximate entropy' properties can better analyse the financial time series than other entropy measures. Therefore, this study used it to quantify the original time series's predictability degree, representing the return rates of different stock markets with different time scales. Then compared the results with the ones we obtained using both $r_i^{int}(t)$ and $r_i^{ext}(t)$. The $ApEn$ parameter r has been fixed to a recommended value equal to 0.2* standard deviation of the series of data under analysis (literature considered it a standard value (Chou, 2014)). In contrast, the embedding dimension is fixed to a widely validate value m=2 Pincus (2008).

3   Empirical results

The external trend and internal component analysis decomposition method have some conditions assumed for the data. This study used only the stocks that fulfilled the following:

- Internal fluctuations and the global trend are statistically independent.

- From stock to stock, correlations between the local fluctuations are negligible.

After decomposition, one of the most important conditions is that the prefactor a_i does not vary over time. Its stability has been checked and confirmed for all the stock used in this study ($\mu = 1$ is the harmless choice to make as it has been mentioned in (Barthélemy et al., 2010)). Once this condition is fulfilled, the approximate entropy technique has been applied to different quantities. The results were compared to determine whether $r_i^{int}(t)$ and $r_i^{ext}(t)$ have smaller approximate

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entropy values than $r_i(t)$. In this study, the $ApEn$ of the S&P 500 daily stocks were calculated using 3 different parameter values from Table 1. The results were similar for the three different values, which means that the predictability is independent of this parameter.
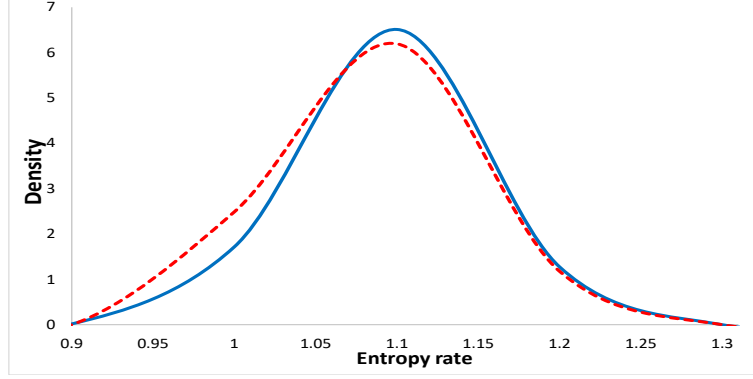

Figure 1: **Kernel density for S&P500 daily entropy rates ($ApEn$).**

Figure 1 presents the kernel density of $ApEn$ rate estimates for both $r_i(t)$ (solid line) and $r_i^{int}(t)$ (dashes line). It can be seen that generally, the entropy rates of the $r_i^{int}(t)$ are lower than $r_i(t)$'s entropy rates, which means that they are more predictable. These values are calculated using different embedding dimensions to explore the impact of m parameter choice (m=2 or 3 because higher embedding dimensions are rarely used in practice). As shown in Table 2, $r_i^{int}(t)$and $r_i^{ext}(t)$ have smaller approximate entropy estimated averages than $r_i(t)$. From this, it can be concluded that both of the quantities $r_i^{int}(t)$and $r_i^{ext}(t)$ are more predictable than $r_i(t)$. Additionally, the embedding dimension m affects on the results ($ApEn$ values have slightly changed in Table 2).

As stated earlier, $ApEn$ is applied to data from NASDAQ 100, SSE and SZSE

Table 2: **Estimated *ApEn* averages for different embedding dimension**

/	m=2	m=3
$r_i(t)$	1.63	1.08
$r_i^{int}(t)$	1.60	1.07
$r_i^{ext}(t)$	1.61	0.94

500 composite indices, respectively (m=2, r=0.2). Figure 2 represents some of their kernel densities and table 3 shows the estimated averages.

Table 3: **Estimated *ApEn* averages of data from Nasdaq 100(3 days data), SSE(weekly data) and SZSE composite(monthly data).**

/	SSE	NASDAQ 100	SZSE 500
$r_i(t)$	1.56	1.36	0.58
$r_i^{int}(t)$	1.55	1.34	0.56

As shown in Table 3, the approximate entropy averages of $r_i^{int}(t)$ are smaller than $r_i(t)$. Figure 2 shows the entropy rates of the $r_i^{int}(t)$ are lower than $r_i(t)$'s entropy rates. From these table and figure, it can be concluded that predictability degree of internal components has improved.

It has been mentioned above that the a_i's are stable, which means that each stock reacts in the same way to the common collective trend $w(t)$ over time. Therefore, it is reasonable to obtain equal approximate entropy values for the external parts of all the stocks belonging to the same stock market. Their estimated averages for the same stock market indices mentioned earlier are equal to 1.54, 1.27 and 0.38, respectively smaller than 1.56, 1.36 and 0.58(estimated averages of $r_i(t)$'s rates). Based on these results showing that both $r_i^{int}(t)$ and $r_i^{ext}(t)$ are more predictable than $r_i(t)$, it is possible to conclude that decomposing financial
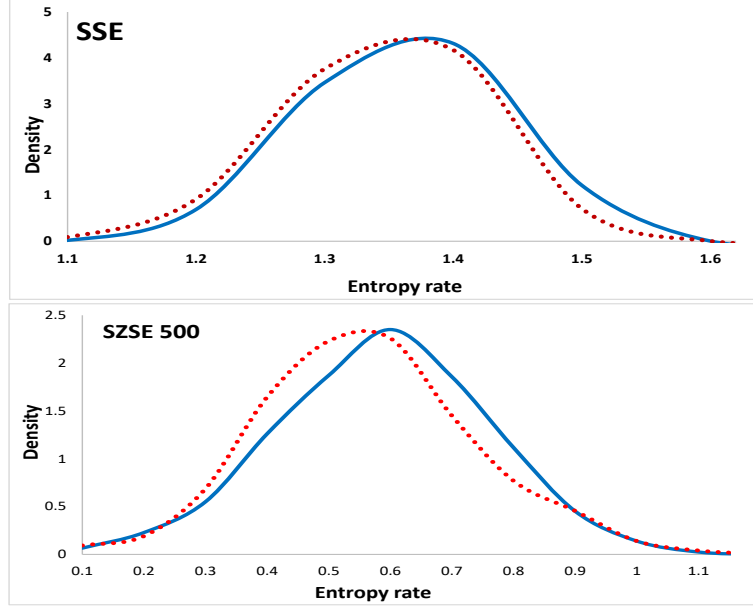
Figure 2: **Kernel density for weekly and monthly entropy rates of SSE index and SZSE 500 composite indices, respectively.**

time series via ETICA is effective to decrease the *ApEn*, hence improve the predictability degree.

The mix in empirical evidence of returns predictability suggested in many studies that it is better to predict their sign instead. Diebold and Christoffersen have developed this area in their theoretical work (Christoffersen and Diebold, 2006) and demonstrated that returns' sign is predictable. Their model has been extended to offer investors the highest gains (Chronopoulos et al., 2018). Therefore, this study considered the consequences of decomposing the absolute value of returns instead of the returns. The ETICA has been applied to $|r_i(t)|$ (to obtain *int abs* and *ext abs*).
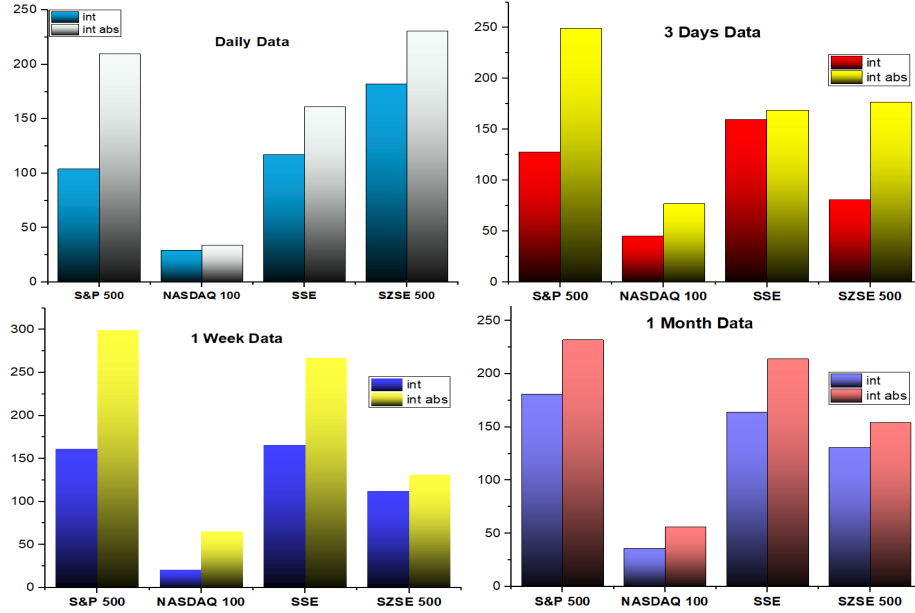
Figure 3: **Number of stocks having internal components approximate entropy smaller than the approximate entropy of the returns.**

Figures 3 and 4 show the number of stocks having improved predictability after decomposing $r_i(t)$ and $|r_i(t)|$ (for both internal and external components). The data are daily, 3-days, weekly, and monthly from S&P 500, NASDAQ 100, SSE and SZSE 500 composite indices. As it can be seen from these histograms, the number of stocks has always augmented in the case of decomposing $|r_i(t)|$ instead of $r_i(t)$. Hence, utilising the absolute value of returns has enhanced the decomposition performance in predictability improvement. Moreover, using the absolute value of $r_i(t)$ has affected the number of stocks and improves the predictability of it.

This article analysed the data frequency impact on predictability. The $ApEn$ estimated average of minute data from the S&P 500 index were compared with the
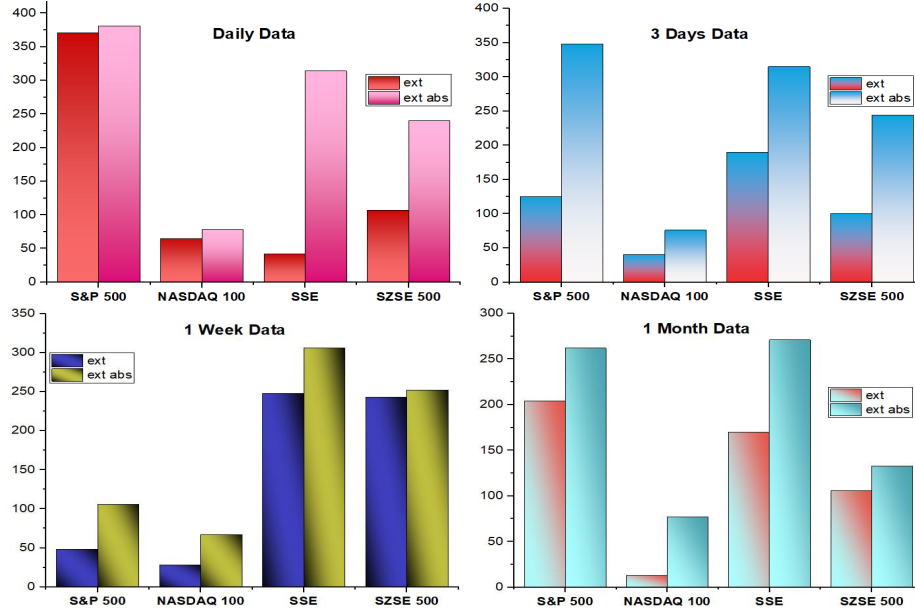
Figure 4: **Number of stocks having external components approximate entropy smaller than the approximate entropy of the returns**

daily averages. The estimated minute data average is recorded as 1.27(m=2) and 0.95(m=3), while the daily averages are 1.63(m=2) and 1.08(m=3). The intraday averages are smaller, which means that they are more predictable than daily data, consistent with the empirical findings of Fiedor for the NYSE 100 stocks (Fiedor, 2014a).

4   Conclusion

This paper used the external trend and internal components analysis decomposition method to increase financial returns predictability. The outcomes show that returns become more predictable after separating the local components from the external trend. Furthermore, decomposing the returns absolute value instead of

returns can enhance its performance. This study examines the effect of frequency on predictability. The results show that intraday data are more predictable than daily data. The findings can guide studies dealing with forecasting models to improve the returns directional prediction correctness.

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