

MAXWELL'S ARTFUL ENTANGLEMENT OF OPTICS AND ELECTROMAGNETISM AND THE INCOMMENSURABILITY TENET

Rinat M.Nugayev¹ and Rinat M Nugayev^{2,3}

¹Affiliation not available

²Region State University of Physical Culture, Sport and Tourism

³Universiade Village 35, the Republic of Tatarstan

April 25, 2024

Abstract

It is contended that one of the promising directions for brooding over the problem of incommensurability of paradigms, coined by T. Kuhn and P. Feyerabend, can be associated with the trend of advancement of neo-Kantian epistemology, embodied by the writings of Ernst Cassirer. According to Cassirer, the statements fixing connections and relationships between mathematical ideal constructs form a 'neutral language' that can serve as a firm ground for comparing the 'old' and 'new' paradigms. A case study of the genesis and functioning of a neutral mathematical language related to the Maxwellian celebrated synthesis of optics and electromagnetism is provided. It is elicited that its basis is constituted by the mathematical language of continuum mechanics. The main function of the neutral language was to project the consequences of all the unified theories onto the mathematical model, 'rewrite' all known laws in this mathematical language, compare their conclusions with each other to eliminate contradictions and generalize the stuff in a self-consistent system of equations.

MAXWELL'S ARTFUL ENTANGLEMENT OF OPTICS AND ELECTROMAGNETISM AND THE INCOMMENSURABILITY TENET.

Prof. Dr. Rinat M. Nugayev, Volga Region State University of Physical Culture, Sport and Tourism, 420138 Kazan, Universiade Village 35, the Republic of Tatarstan, the Russian Federation.

Telephone: +79518968538.

Home address: 420043 Kazan, Chekhov str., building 53, apt. 62, the Republic of Tatarstan, the Russian Federation.

rinatnugaev@mail.ru

nugayevrinat@gmail.com

ORCID ID: 0000-0001-9925-2264

Abstract. It is contended that one of the promising directions for brooding over the problem of incommensurability of paradigms, coined by T. Kuhn and P. Feyerabend, can be associated with the trend of advancement of neo-Kantian epistemology, embodied by the writings of Ernst Cassirer. According to Cassirer, the statements fixing connections and relationships between mathematical ideal constructs form a 'neutral language' that can serve as a firm ground for comparing the 'old' and 'new' paradigms.

A case study of the genesis and functioning of a neutral mathematical language related to the Maxwellian celebrated synthesis of optics and electromagnetism is provided. It is elicited that its basis is constituted by the mathematical language of continuum mechanics. The main function of the neutral language was to project the consequences of all the unified theories onto the mathematical model, 'rewrite' all known laws in this mathematical language, compare their conclusions with each other to eliminate contradictions and generalize the stuff in a self-consistent system of equations.

Keywords: incommensurability, Cassirer, mathematization, Maxwell, unification, electromagnetism.

1.Introduction.

In the process of the formation of the post-positivist stage of the evolution of the philosophy of science, Thomas Kuhn and Paul Feyerabend had formulated and substantiated the well-known thesis on the incommensurability of paradigms replacing each other in the processes of scientific revolutions (the ‘incommensurability tenet’). According to this thesis, the creators of new mature scientific theories (Newton, Maxwell, Einstein) make such profound transformations of theoretical ontologies, so resolutely reject obsolete ideas that the ‘new’ paradigms they construct become incommensurable with the ‘old’ ones. This means that in science there are no objective bases for comparing successive paradigms.

Thus, in his epoch-making “The Structure of Scientific Revolutions”, T. Kuhn provides a classic example of incommensurability related to the process of replacing Newtonian mechanics with special relativity:

“But the physical referents of these Einsteinian concepts are by no means identical with those of the Newtonian concepts that bear the same meaning. (Newtonian mass is conserved; Einsteinian is convertible with energy. Only at low relative velocities may the two be measured in the same way, and even then they must not be conceived to be the same [...]) The normal-scientific tradition that emerges from a scientific revolution is not only incompatible but often actually incommensurable with that which has gone before” (Kuhn [1962], 1970, pp. 102-103).

Paul Feyerabend adhered to a similar, although less rigid and more structured position (Feyerabend [1970], 2010). The so-called ‘incommensurability problem’ consists in that if the ‘new’ and ‘old’ paradigms are really incommensurable, then on what grounds can we argue that the ‘new’ paradigm is better than the ‘old’ one? And if we cannot assert this, then what is the progress in the development of science?

Moreover, since the ‘old’ and ‘new’ paradigms are radically separated from one another by the stiff barrier of ‘incommensurability’, it is difficult to see how one gets from one to the other. “Kuhn faces a severe difficulty explaining how new ideas emerge against the background of their predecessors, from which they are so totally distinct” (Renn 1993, p.312).

Forsooth,

“the new paradigm, or a sufficient hint to permit later articulation, emerges all at once, sometimes in the middle of the night, in the mind of a man deeply immersed in crisis. What the nature of that final stage is – how an individual invents (or finds he has invented) a new way of giving order to date now all assembled – must have remain inscrutable and may be permanently so” (Kuhn [1962], 1970, pp.89-90).

Furthermore,

“There are many things to be said about the nature of progress in the sciences; the thing that you *cannot* I think say coherently is that they get closer and closer to the truth... [Hence] That’s what you do within a system, - judge the truth or falsity of statements. Across a system you can’t apply that sort of calculation... [Thus] I take theories to be whole systems, and as such they don’t need to be true or false” (Thomas Kuhn ‘s interview to Skuli Sigurdsson, 2017, p.24).

Therefore, of undoubted interest for the modern philosophy of science are those concepts that precede the study of specific problems of the philosophy of science by the adoption of a general view of the evolution of science and, accordingly, clearer criteria for scientific progress. This, as a rule, is possible when this view is formulated as a continuation of one or another classical philosophical tradition. It makes possible to apply the well-structured apparatus of classical epistemology, which, in particular, makes it possible to more clearly distinguish ‘truth’ from ‘error’. Such a promising standpoint is the epistemology of Ernst Cassirer, advanced in line with the neo-Kantian philosophical tradition. The latter, as part of the ongoing rejection of neo-positivist concepts, is attracting more and more attention of modern researchers (Ryckman 2005; Ladyman 2008, et al). The purpose of this chapter is to analyze the concept of Cassirer from the point of view of the problem of (in) commensurability (first and second sections) and to show, using the vivid example of the Maxwellian revolution, what possibilities it provides for solving this problem (third section).

2.Neo-Cantian Epistemology of Ernst Cassirer.

In accordance with the traditions of the Marburg school of neo-Kantianism, the fundamental concepts of each science, the means by which it raises its 'ultimate' questions and formulates certain conclusions, are not passive reflections of human existence, but ‘intellectual symbols’ constructed by people themselves (Cassirer 1923). And it was physical and mathematical cognition that most clearly realized the symbolic nature of its fundamental means of knowledge before the others. According to Cassirer, modern science arose only in the XVth - XVIIth centuries. Only when two diverse research traditions that had previously developed relatively independently of each other met and began to ‘interpenetrate and actively interact’ in Western European culture.

The first tradition (Aristotle) is the tradition of experimental natural science, according to which all scientific knowledge arises out of experience. The second tradition (Euclid) is the tradition of mathematically immaculate reasoning, according to which every scientific statement must be proven, i.e. in logically impeccable way derived from the system of self-evident axioms.

Interaction of these traditions constituted one of those antinomies, the unsolvability of which continues to develop natural science.

The result of how “the fundamental forms of knowledge penetrate into each other - sensory sensation and pure contemplation, categories of pure reason and ideas of pure reason; how the theoretical image of reality is determined by their interconnection and interaction” [Cassirer 1923, p.14] is the construction of systems of ideal mathematical objects. Thus, the laws of classical (Newtonian) mechanics make sense only when they describe connections and relationships between ‘ideal ultimate images’, which we abstractly put in place of empirical sense data of sense perception. Accordingly, mechanical movement is a predicate that cannot be directly applied to the objects of the sensory world around us. It makes sense "only when applied to the class of objects that mathematics substitutes in their place in its free creativity" [Cassirer 1910, p.143].

Being radically opposed to the traditional logical teaching of Aristotle, the path of mathematical formation of concepts was blazed by Galileo, Kepler and Newton 'through the method of forming series'. The latter does not at all consist in eliciting from a certain throng of homogeneous impressions what is common to them, but in establishing

“a certain principle by virtue of which the different follows one from the other. The unity of the concept was found not in some unchanging sum of attributes, but in the rule, due to which a simple difference was presented as a regular series of elements” (Cassirer 1910, p.172).

As a result, in place of single 'values of the series', established by observation or measurement, one puts "limiting values towards which the series as a whole move." Natural science theories provide illustrative examples of the application of this constructive method of ‘passing to the limit’: a perfectly elastic body, an ideal gas, an incompressible fluid, a perfect circular process, etc. Classical mechanics itself describes the strict laws of motion only through the use of the ultimate idea of a ‘material point’.

But, in introducing this ‘realm of ultimate ideal objects’, we must in no way confuse it with the world of Platonic ideas or forms described by Kepler:

"Mathematical ideas are the eternal prototypes and 'archetypes' according to which the Divine Builder of the world arranged everything."

As Cassirer himself never tired of repeating,

“it is no longer possible to be mistaken and take the objects of physics - mass, force, atom, ether - as new realities that must be explored and inside of which one must penetrate, once we understand that they are tools created by thought to depict the chaos of phenomena in the form of a dismembered and measurable whole” [Cassirer 1910, p.193].

Using numerous examples described in detail in the works of the great German physicist Heinrich Hertz, Cassirer concludes that the fundamental concepts of the natural sciences, and especially the basic concepts of mechanics and electrodynamics, are by no means copies of things directly given in the experience. On the contrary, they are the necessary ‘constructive blueprints’ of physical thought.

The theoretical significance of these projects is primarily determined by the extent to which the consequences derived from them coincide with the experimental results obtained. On the other hand, any concept of natural science, in the specific meaning that it has in mathematics and physics, is

“merely a symbol for a certain form of connection, which has thrown off one by one any special material content. It means only the kind of possible coordination, but not the ‘essence’ (das Was) of the elements that must be coordinated with each other (Cassirer 1910, p.236).

As a result, instead of conjuring up a new truly Platonic world behind the world of perceptions, our knowledge limits itself to sketching out the general logical schemes with which it expects to represent the connections and relations of perceptions. A natural-science symbol has its correlate not in the constituent parts of perception, but in the regular *connection* that exists between its parts.

3. THE PLACE OF THE CONCEPT OF SCIENTIFIC PROGRESS IN THE EPISTEMOLOGY OF ERNST CASSIRER.

In accordance with the entrenched Kantian tradition, Cassirerian epistemology proceeds from the fact that modern science categorically rejects natural philosophical claims to penetrate ‘inside’ nature, implying its last substantial foundations. The development of science appears in the Cassirer version of neo-Kantian epistemology as follows.

I. First, a careful differentiation is made of the individual spheres of perception offered to us by empirical contemplation; in each of them, ‘fluid transitions’ are replaced by precise, well-fixed numerical definitions. At the same time, the ‘world of physics’ (however, like the ‘worlds’ of chemistry, biology, etc.) is subdivided and articulated according to differences directly given in sensation. Optics corresponds to light and color sensations, thermodynamics corresponds to thermal sensations, and acoustics corresponds to sound sensations. But already at this stage, in order to enter the theoretical schemes of optics, thermodynamics, etc., the sensory content must go through the transformations described above, when certain sensations are replaced in the primary schemes by certain limits of mathematical sequences or, in Russian vocabulary (Stepin 2006) “particular theoretical schemes ”.

II. But inevitably comes the second, more significant stage.

“The genuine and much more difficult task of physics is not to pass from sensible ‘qualities’ to precisely defined quantities. Rather, after the implementation of this preparatory work, she raises the really fundamental question for her about *the functional connection* and connection of individual qualitative areas. They should not be made out in simple coexistence and succession, but they should be thought of as a law-like and law-governed unity” (Cassirer 1923, p.340).

To illustrate this thesis, Cassirer refers to Max Planck's famous report "The Unity of the Physical Picture of the World". In it, the founder of quantum physics convincingly substantiates the thesis according to which true progress in the natural sciences consists in the ever greater *liberation* of theoretical thought from the restrictions imposed on it by the initial attachment to the direct content of sensory sensations and to the primitive way of their articulation, i.e. in a decisive 'emancipation from anthropomorphic elements'. In particular, the advancement of physics and chemistry did not stop at a purely empirical establishment of the existence of various constants, but moved on to the establishment of general systemic relationships that made the appearance of certain complexes 'understandable'.

This 'comprehensibility' essentially meant the search for some universal laws that determine the connections of heterogeneous constants. With the help of these laws, areas that were previously considered extremely distant from each other began to be considered from a single point of view. At first, this approach was limited to certain areas, but then it became the 'main trend of modern physics'. First, the constants describing physical and chemical processes approached each other. But a closer convergence of the constants occurred only in the middle of the XIXth century, when James Maxwell revealed the relationship between the constants that determine the optical properties of certain substances and the constants that characterize their electrical conductivity.

At the same time, the 'general schematism of the concept of number' plays a decisive role in the evolution of the second stage. Namely:

"Number functions as a kind of *abstract medium* in which different sensory areas meet each other, losing their sensory heterogeneity in the process. For example, in Maxwell's theory, the phenomenon of light is identified with the phenomenon of electricity, since they receive the same numerical value and expression. The external difference between optical and electrical phenomena disappears as soon as it is recognized that the constant c , appearing in Maxwell's equations, is exactly equal to the speed of light in vacuum. It is the form of a purely numerical relation that helps to overcome the heterogeneity of sensory properties and achieve the homogeneity of the physical 'essence'" [Cassirer 1923, p.346].

Note that the leader of German theoretical physics at the beginning of the XXth century, cited above, came, in the course of reflection on the foundations of his professional activity, to similar conclusions (Planck 1922). Returning to the Maxwellian synthesis, one can state that, as its most important result, it was recognized that all the laws valid for light rays (refraction, interference, polarization, etc.) are equally applicable to thermal radiation. Thus, a strong 'union' was ascertained between these two areas, which made it possible to overcome the qualitative difference in sensations in which warmth and light are given to us.

From now on, they began to differ - in the 'objective sense' of physical judgments - only by numerical values, only by a certain parameter denoting the 'wavelength' of the two varieties of electromagnetic radiation.

Eventually

"the consistent development of a genuinely physical form of thinking and its symbolism gave rise to a [neutral!] language that created the possibility of such an unheard-of expansion, which disclosed that the boundaries of sensation are merely random 'anthropomorphic' ones" (Cassirer 1923, p. 347).

But the most apparently considered feature of the evolution of the second stage was manifested in the history of the discovery of the law of conservation of energy in the formulation of Robert Mayer. As Cassirer emphasized, for the latter, the mechanical law of conservation of active force means nothing more than

"a universal relation connecting various areas of physical phenomena and making them *qualitatively comparable and commensurable* [...] The principle establishes fixed numerical relations in the transformation of heat into motion and motion into heat, but it does not in any way surmise that heat in its essence is nothing else, as movement" (Cassirer 1923, p.353).

And, finally, as a result of the joint successes of Maxwell, Lorentz and Einstein, the theory of the electromagnetic field and the special theory of relativity, the ether, as 'wooden iron', amalgamating mutually exclusive features, was discarded, and such a reality as the 'physical field' completely reigned. But at the same time

"the reality we call 'field' is no longer conceivable as a complex of physical 'things', but it is expressible as a set of physical relations" (Cassirer 1923, p. 365).

The 'essence' of light no longer consists for us in the 'movement' of a wave or in 'oscillations', but consists only in the periodic changes of a vector, whose direction is always considered to be perpendicular to the direction of the light. Progress in the development of physics does not at all consist in a deeper immersion into the essence of the processes under consideration, expressed in discovering, finally, the true 'substance' from which all things are made. It consists in combining various physical theories, in bringing the integrity of natural phenomena under one universal rule that establishes connections and relationships between various sections of physics (Nugayev 2020).

And the result of this unification should be a new system of relations describing the connections between already known idealizations of physical processes. Thus, the process of unification of scientific theories turns out to be inextricably linked with another deeper process expressing the progress of scientific knowledge. As a result of the unification of physical theories

“With the individual peculiarity of impressions, its inner heterogeneity has also disappeared, so that regions that, from the point of view of sensation, are completely incomparable with each other, can now be understood as interconnected members of the same general plan. Only in this lies the special value of scientific constructive building; in it turns out to be connected by means of continuously stretching intermediate logical terms that which in the first naive view is alien and unrelated to each other” (Cassirer 1923, p.352).

4. A Case Study of Neutral Language Functioning: Maxwell’s Masterful Synthese of Optics and Electromagnetism.

It is a commonplace that Maxwellian electrodynamics was a stage of the Faraday program advancement based on the field concept (see, for instance, Chalmers, 1975 and references cited therein). The latter had provided prediction and verification of startling electromagnetic waves phenomenon and have superseded at last the Ampere-Weber research program based on the action at a distance concept.

However a more pervasive account of the XIX-th century physics that became possible due to the studies of Daniel Siegel (1991), Margaret Morrison (2000) and Olivier Darrigol (2001) enables one to challenge this point of view as an oversimplification – and to provide a modified version of it – with a help of the following reasons.

(1) At first, James Maxwell himself many times, beginning from his first paper and up to the last one, had stressed that the pivotal ideas of the Ampere- Weber electrodynamics were *complimentary* and *not* alternative to those of the field concept. Even starting his electrodynamics studies, on May 1855, a post-graduate student at Cambridge wrote to his father

“I am working away at electricity again, and have been working my way into the views of heavy German writers. It takes a long time to reduce to order all the notions one gets *from* these men, but I hope to see *my way* through the subject, and arrive at something intelligible in the way of a theory” (quoted from Campbell l& Garnett, 1882, p.105).

And, at last, in his epoch-making “*Treatise on Electricity and Magnetism*” Maxwell pictures contrivance of his system of equations in the following way:

“I was aware that there was supposed to be a difference between Faraday’s way of conceiving phenomena and that of the mathematicians, so that neither he nor they were satisfied with each other’s language. I had also the conviction that this discrepancy did not arise from either party being wrong” (Maxwell [1873a], p.499).

(2) No-one at Cavendish laboratory - conducted and well-equipped by Maxwell - had made a serious and sustained attempt to confirm Maxwell’s theory (Mahon, 2003). Maxwellian electrodynamics’ main verification was provided by a star student of Hermann Helmholtz whose name was Heinrich Hertz (1888). And the latter did not consider himself a follower of Maxwell (Darrigol, 2001).

(3) Faraday's influence on Maxwell was strongly exaggerated. The explanation of the field concept adoption due to the sympathy to intermediate action notion is not confirmed by Maxwell's papers more thorough analysis (Shapiro, 1973). It reveals that Maxwell started to take the field notion as a part of physical reality sufficiently late: only *after* he had derived the electromagnetic waves existence from his equations, i.e. *after* the derivation of the "displacement current". Up to that moment he had used the field notion only as an illustrative means for building up the pictorial images of complicated vector differential equations.

For instance, within one and the same paper Maxwell (1856) uses different models for illustrating different analytic relations, punctuating that:

"It is not even a hypothetical fluid which is introduced to explain actual phenomena. It is merely a collection of *imaginary* properties which may be employed for establishing certain theorems in pure mathematics in a way more intelligible to many minds and more applicable to physical problems than that in which algebraic symbols alone are used" (Maxwell [1856], 1890, p.160).

The aim of the present chapter is to make a further step in proffering such a rational reconstruction of Maxwellian electrodynamics genesis that takes into account (1) – (3). The reconstruction should provide a "theoretically progressive problemshift" relative to other reconstructions contending that Maxwellian revolution is a more complex phenomenon than appears from the standpoints of some well-known scientific revolution models (Kuhn, 1977; Lakatos, 1978).

I had already underscored that one of these models' drawbacks consists in that they lack the mechanisms of the paradigms' (or scientific research programmes') *interactions* (Nugayev, 1999; 2012). And it is one of my basic aims to exhibit that the remark is especially appropriate for Maxwellian electrodynamics genesis. I'll try to indicate that the Maxwellian program had superseded that of Ampere-Weber only because it had constantly communicated with it. The Maxwellian program did embrace some of the propositions of the Ampere-Weber "hard core", as well as some surmises of the Faraday and Young-Fresnel programs.

In my view the main drawbacks of the preceding studies consist in *underestimation of Maxwell's own methodology* created by himself for his ambitious project of mechanics, electrodynamics and optics unification. Though in every domain of creativity (including epistemology) Maxwell always took his own way; and he tried to teach his students in the same way too. Recall the following passage of his Marichal college speech:

"It is best that every man should be settled in *his own* mind, and not be led into other men's ways of thinking under the pretense of studying science" (quoted from Mahon, 2003, p.70).

As the author of ground-breaking “*Treatise on Electricity and Magnetism*” himself put it in one of his letters, “I find I get fonder of metaphysics and less of calculations continually” (quoted from Campbell & Garnett, 1882, p. 298). One can recall Gustav Kirchhoff: “He is a genius, but one has to check his calculations”.

In my view one should take Ludwig Boltzmann’s comments on Maxwell’s works more seriously. In immaculate lectures on Maxwell’s theory as well as in his comments on Maxwell’s electrodynamics papers (that he had translated into German), the founder of statistical mechanics had stressed that many Maxwell’s works but especially his early electrical papers “were not properly understood”. Probably it can be explained by the fact that these works ‘written according to the *long-term plan*’ disclose that their author “was as mastermind in theory of knowledge as he was in the field of theoretical physics” (Boltzmann, 1895). Maxwell’s neo-Kantian epistemology that sprung out from an intention to find a fruitful compromise between the extremes of Kantian relativism and Scottish ‘common sense realism’ was a necessary part of his ambitious unification of optics and electromagnetism design.

Withal, Maxwell was not the first to unify optics and electromagnetism. Yet he did not like the way his predecessors had done it. Why?

“The present state of electrical science seems peculiarly unfavorable to speculation...No electrical theory can now be put forth, unless it shows the *connexion* not only between electricity at rest and current electricity, but between the attractions and inductive effects of electricity in both states...The results of this simplification may take the form of a purely mathematical formula or of a physical hypothesis. In the first case we entirely lose sight of the phenomena to be explained; and though we may trace out the consequences of given laws, we can never obtain more extended views of the *connexions* of the subject” (Maxwell, [1856], p.155).

His eminent predecessors were Hans Christian Oersted (1777 – 1851), Andre-Marie Ampère (1775-1836), Wilhelm Weber (1804-1890), Michael Faraday (1791-1867) and William Thomson (1824-1907). Yet Maxwell’s Weltanschauung was characterized by *an extraordinary high level of philosophical culture*. A brilliant student at Edinburgh and Cambridge and a post-graduate at Cambridge was enchanted by a profound skepticism of David Hume, George Berkley and especially Immanuel Kant at the lectures of Sir William Hamilton on mental philosophy at Edinburgh University.

For instance, in the 03.25.1854 frank letter Maxwell states that

“I have been reading Berkeley on “The Theory of Vision”, and greatly admire it, *as I do all his other non-mathematical works*; but I was disappointed to find that he had at last fallen into the snare of his own paradoxes”(quoted from Campbell& Garnett, 1882,p.109).

Likewise,

“Comte has good ideas about method, but no notion of what is meant by a person” (Campbell &Garnett, 1882, p.108).

Hamilton's dazzling lectures, which were a prominent component of the Scottish university curriculum, 'interested him greatly'. From the Class of Metaphysics his mind 'gained many lasting impressions' (Lewis & Campbell, 1882). The conspicuous lectures of Sir Hamilton made a strong impression on him, in "stimulating the love of speculation to which his mind was prone" (Niven, 1890).

Sir William Hamilton (1788-1856) was one of the most outstanding representatives of Scottish 'common sense philosophy', a worthy heir of Thomas Reid and James Stewart. Yet it is no coincidence that he had visited Germany two times and stressed *Kant's proposition that all knowledge is relative*; so we know nothing about things themselves except by their *relationship* to other things. Correspondingly, he had stimulated a spirit of criticism in his pupils by insisting on the great importance of psychology as opposed to the older metaphysical method.

Hamilton's 'philosophy of the conditioned' surely had an unequivocal Kantian flavor. Like Kant, he held that one can have knowledge only of "the relative manifestations of an existence, which in itself it is our highest wisdom to recognize as beyond the reach of philosophy". Though unlike Kant *he had argued for the position of a 'natural realism'* in the Reidian tradition.

The Reverend Thomas Reid (1710-1796) directed his thought-provoking "*An Inquiry into the Human Mind on the Principles of Common Sense*" (1764) against Hume and Berkeley. It is here – he asserted – that the 'danger of the idealism' lies – in its reduction of reality to "particular perceptions", essentially *unconnected* with each other. The independent unit of knowledge is not an isolated impression but a judgement. Yet in such a judgement is contained the reference both to a permanent subject and to a permanent world of thought, and, implied in these, such judgements, for example, as those of existence, substance, cause and effect. These principles are not derived from sensations, but are 'suggested' on occasion of sensation, in such a way as to constitute the necessary conditions of having perceptive experience at all.

The sober doctrine of relativity of knowledge has seemed to many – including James Stewart Mill – contradictory to his realism. But for Hamilton, the two are held together by a kind of intuitionism that emphasizes certain facts of consciousness that are both primitive and incomprehensible. They are though constitutive of knowledge, "less forms of cognitions than of beliefs" (quoted from Audi, 1999, p.360).

The subtle relativism or phenomenalism which Hamilton adopted from Kant and sought to engraft upon Scottish philosophy is absent from the original Scottish doctrine. Denying Hume's skepticism, Hamilton did his best to *find a compromise* between Kant's relativism and Reid's realism; and it was namely that that Maxwell have pointed out as a basic point of his metaphysical program on moving from Edinburgh to Cambridge:

“in the meantime I have my usual superfluity of plans...4. Metaphysics – *Kant’s Kritik of Pure reason in German, read with a determination to make it agree with Sir W. Hamilton...*”(Campbell & Garnett, 1882,p.77).

The ‘Copernican revolution’ in epistemology that had been initiated by Kant consisted in that the world of common every-day experience (i.e. Husserl’s ‘*lebenswelt*’) had lost its dominating position in interpreting things that can be perceived by our senses. Kant had exchanged the world of common experience by the world of Galilean experimental and mathematical physics grounded on the idealizations of the ‘*lebenswelt*’ phenomena. Hence truth became not only something spontaneously revealing and disclosing itself but something that can be comprehended only by a special (‘scientific’) method.

On the other hand, if truth is comprehended only in experience and one can grasp not ‘the things by themselves’ but just the ‘phenomena’, it is necessary to decidedly reject the very opportunity of reaching the absolute truth. Our sensory representation is by no means a representation of things ‘in themselves’, but merely of the way in which they appear to us. Hence the so-called ‘*analogies of experience*’ are especially important in Kant’s epistemology.

“This type of cognition is cognition *according to analogy*, which surely does not signify, as the word is usually taken, an imperfect similarity between two things, but rather a perfect similarity between two relations in wholly dissimilar things. Such is an analogy between the legal relation of human actions and the mechanical relation of moving forces: I can never do anything to another without giving him a right to do the same to me under the same condition; just as a body cannot act on another body with its motive force without thereby causing the other body to react just as much on it. Right and motive force are here completely dissimilar things, but in their relation there nonetheless complete similarity. By means of such an analogy I can therefore provide a concept of a relation to things that are absolutely unknown to me “(Kant [1783], 2002, p.146-147).

In more detail, in ‘*The Critique of Pure Reason*’, Kant elucidates a more interesting example:

“Prior to the perception, however, and therefore completely a priori, we are able to cognize its existence, provided it stands in connection with some perceptions according to the principles of the empirical conjunction of these, that is, in conformity with the analogies of perception. For, in this case, the existence of the supposed things is connected with our perception in a possible experience, and we are able, with the guidance of these analogies, to reason in the series of possible perceptions from a thing which we do really perceive to the thing we do not perceive. Thus, we *cognize the existence of a magnetic matter penetrating all bodies from the perception of the attraction of the steel-fillings by the magnet*, although the constitution of our organs renders an immediate perception of this matter impossible for us” (Kant [1787], 2010, p.170).

Note that *even the example of the analogy of experience was taken by Kant from the domain of electromagnetism* thus paving the way to Maxwell. The latter had punctuated many times that things we can measure directly, like mechanical force, are merely the outward manifestations of deeper processes, involving entities like electric field strength, which are beyond our power of visualization (see Mahon, 2003).

A more detailed exposition of Maxwell's research program that he had followed through all his life is provided by his following truly philosophical works – a speech “*Are There Real Analogies in Nature?*” read at the ‘Apostles’ Cambridge club in 1856 (just after the publication of his thorough paper “*On Faraday's Lines of Force*”, 1855-1856) – and his stupendous paper “*Helmholtz*” (1877).

Discreet Cambridge speech is not a crude exposition of Kant's epistemology but a tense discussion of Maxwell with ‘Kant in himself’. It is not accidental that its very heading contains a question and not a proposition: “*Are There Real Analogies in Nature?*” Maxwell did not give a definite answer - in full accord with Kant's antinomies that represent human Reason's attempts to overstep the limits of Experience. He multiplies arguments pro and contra the assertion that there certainly *are* real analogies in Nature.

He commences the speech contending that

“the whole framework of science, up to the very pinnacle of philosophy, seems sometimes a dissected model of nature, and sometimes a natural growth of the inner surface of the mind” (Campbell& Garnett, 1882, p.133).

Or in the other part of the speech:

“are we to conclude that these various departments of nature in which analogous laws exist have a real interdependence; or that their relation is only apparent and owing to the necessary conditions of human thought?” (Campbell& Garnett, 1882, p. 134).

Certainly Maxwell's thinking in terms of Kantian antinomies is only a facet of his approach. In Hamilton's wake, Maxwell tries to find a way of *his own* between the Scylla of Kantian transcendentalism and the Charybdis of Scotch common sense realism.

In modern times the Scottish view of knowledge is characterized by the following principles (Mertz, 1964; Olson, 1975).

1. All knowledge is relational.
2. Analogies (and models connected with them) are among the chief such relational models of knowing.
3. Analogies and models are necessary for psychological reasons. For common people, understanding requires the use of models for simplifying and organizing knowledge.
4. Strong psychological tendencies in the Scottish Common Sense tradition admit reconciliation with logical and analytical components of Kantian philosophy.

Hence for Maxwell the effective resolution of the antinomies came from adopting *partial points of view*, as all human knowledge is partial. What remains is establishing correspondences or analogies.

“Whenever they see a relation between two things they know well, and think they see there must be a similar relation between things less known, they reason from the one to another. This supposes that although pairs of things may differ widely from each other, the relation in the one pair may be the same as that in the other. Now, as in a scientific point of view the *relation* is the most important thing to know, a knowledge of the one thing leads us a long way toward a knowledge of the other. If all that we know is relation, and if all the relations of one pair of things correspond to those of another pair, it will be difficult to distinguish the one pair from the other, although not presenting a single point of resemblance, unless we have some difference of relation to something else whereby to distinguish them. Such mistakes can hardly occur except in mathematical and physical analogies...Perhaps the ‘book’, as it has been called, of nature is regularly paged; if so, no doubt the introductory parts will explain those that follow, and the methods taught in the first chapters will be taken for granted and used as illustrations in the more advanced parts of the course; but if it not a ‘book’ at all, but a magazine, nothing is more foolish to suppose that one part can throw light on another” (Maxwell; quoted from Campbell & Garnett 1882, p. 124).

That is the first lesson taught by Kantian epistemology – (I) “the principle of relativity of scientific truth”. Note that even the examples of the analogies are taken by Maxwell from the ‘Prolegomena’. Hence it is not surprising that the next principle – (II) ‘theory laidness of observation’ – is also extracted from Kant:

“The dimmed outlines of phenomenal things all merge into one another unless we put on the focusing glass of theory, and screw it up sometimes to one pitch of definition and sometimes to another, so as to see down into different depths through the great millstone of the world (Maxwell; quoted from Campbell & Garnett 1882, p. 125).

The importance of the last principle for Maxwell’s methodology cannot be overestimated. In nature all the phenomena are tightly interconnected and merge into one another.

All the differences in theoretical approaches are due to the fact that their authors focus on the multifarious facets and different levels of the phenomena investigated. Hence a theoretician’s task is to provide the special notions to cover the various domains of experience. But where should he discover them? – In experience, via immediate generalizations of the experimental data? – Another piece of Maxwell’s creativity – his keen 1854 letter – makes it possible to look into his thought laboratory:

“It is hard work *grinding out* ‘appropriate ideas’, as Whewell calls them. I think they are coming out at last, and by dint of knocking them against all the facts and half-digested theories afloat, I hope to bring them to shape, after which I hope to understand something more about inductive philosophy that I do at present.

I have a project of *sifting* the theory of light and making everything stand upon definite experiments and definite assumptions, so that things may not be supposed to be assumptions when they are either definitions or experiments” (Maxwell; quoted from Campbell & Garnett, 1882, p.112).

Now it is clear where the ‘appropriate ideas’ come from: they are not the slavish copies of things, but are just the a priori forms by which a hotchpotch of sensations is “brought to order”.

According to Maxwell's distinctive essay "*Has everything beautiful in Art its original in Nature?*" (spring of 1854),

"as the Theoretic and Imaginative faculty is far in advance of Reason, he can apprehend and artistically reproduce natural beauty of a higher order than his science can attain to" (Campbell & Garnett, 1882, p.133).

At first the 'appropriate ideas' are vague and dim; yet in the long run they are 'grinded out' by knocking them with the 'facts', as well as other theories. Though theoretician's task is not only to introduce and furnish the subtle notions 'reflecting' the different facets of the phenomena under consideration, but also to unify the notions in effective synthesis. The stages of such a synthesis are highlighted in Maxwell's chef-d-euvre "*Hermann Ludwig Ferdinand Helmholtz*" that begins as follows:

"Hence the ordinary growth of human knowledge is by accumulation round a number of *distinct* centers. The time, however, must sooner or later arrive when two or more departments of knowledge can no longer remain independent of each other, but *must* be fused into a consistent whole. But though men of science may be profoundly convinced of the necessity of such a fusion, the operation itself is a most arduous one. For though *the phenomena of nature are all consistent with each other*, we have to deal not only with these, but with the hypotheses which have been invented to systematize them; and it by no means follows that because one set of observers have labored with all sincerity to reduce to order one group of phenomena, the hypotheses which they have formed will be *consistent* with those by which a second set of observers have explained a different set of phenomena.

Each science may appear tolerably consistent within itself, but before they can be combined into one, each must be stripped of the daubing of untempered mortar by which its parts have been prematurely made to cohere.

Hence the operation of fusing two sciences into one generally involves much criticism of established methods, and the explosion of many pieces of fancied knowledge which may have been long held in scientific reputation" (Maxwell [1877], 1890, p.592).

This passage is not accidental for Maxwell's creativity. In other writings he himself emphasized the asset of '*cross-fertilization* of the sciences' (Maxwell, 1890, vol.2, p.744) evoking the image of bees pollinating flowers (see Harman 2003 for details). The typical example of 'the daubing of untempered mortar elimination' principle (III) for Maxwell was

"the progress of science in Newton's time [which] consisted in getting rid of the celestial machinery with which generations of astronomers had encumbered the heavens, and thus 'sweeping cobwebs off the sky'" (Maxwell [1873], 1890, p.315; see also Nugayev, 2012)

It should be added that the ideas of Maxwell's 1877 paper were generalized and elaborated by a whole number of scholars (Tisza, 1963; Podgoretzky & Smorodinsky, 1980; Nugayev 1985a; 1999; 2018). As a result, a lucid 'mature theory-change model' was proposed. According to the model, the origins of scientific revolutions lie not in a clash of mature theories with facts, but of 'old' mature theories with each other, leading to contradictions that can be successfully eliminated in a more general theory.

Onwards, on the basis of the model the so-called '*conception of communicative rationality*' that tries to take the socio-psychological peculiarities of theory change into account was proposed. The doctrine enables one to describe the basic paradigms' interaction, as well as the interaction of cognitive and of social parts of the paradigms too. The key role in theory change is played by the proponents of the old paradigms' dialogue that leads to genuine interactions in all the three main dimensions including personal, institutional (scientific community) and cultural (theoretical) aspects. It seems to me that some of the features of the theory-change model and communicative rationality conception resemble the following stages of Maxwell's program realization.

First, "*A Treatise on Electricity and Magnetism*" (1873) was mainly an encyclopedia and a textbook. The basic electromagnetic results were obtained in a sequence of three epoch-making papers: "*On Faraday's Lines of Force*" (1856), "*On Physical Lines of Force*" (1861) and "*A Dynamical Theory of Electromagnetic Field*" (1864).

The first one (Maxwell 1856) is dedicated to elaboration of the '*physical analogies*' method sprung out of Kantian epistemology. The method decidedly rejects the 'ontological' approaches striving for the 'essences' of electrical and magnetic phenomena and proclaiming that 'in reality' electricity and magnetism are sheer 'fields' and not 'action at a distance' phenomena, or vice versa. Maxwell's proposal is to take Faraday's lines of force as a kind of inscrutable tubes filled with ideal incompressible fluid.

"I propose then, [...] ; and lastly to show how by an extension of these methods, and the introduction of another idea due to Faraday, the laws of the attractions and inductive actions of magnets and currents may be clearly conceived, without making assumptions as to the *physical nature* of electricity, or adding anything to that which has been already proved by experiment.

By referring everything to the purely geometrical idea of the motion of an imaginary fluid, I hope to attain generality and precision, and to avoid the dangers arising from a premature theory professing to explain the cause of the phenomena" (Maxwell [1856], 1890, p. 159).

It is crucial for a Kantian that this incompressible fluid has nothing to do with experimentally given reality. The single constrain on the concocted models consists in the demand that the mathematical constructs should not contradict each other. In all the other delicate matters the physical analogies method admits an unlimited freedom of imagination. Even the relentless conservation laws can be broken down!

"There is nothing self-contradictory in the conception of these sources where the fluid is created, and sinks where it is annihilated. The properties of the fluid are at our disposal, we have made it incompressible, and now we suppose it produced from nothing at certain points and reduced to nothing at others" (Maxwell [1856], 1890, p. 162).

And in the other parts of the paper Maxwell exhibits the ways by which the idea of incompressible fluid motion can be applied to the sciences of statical electricity, permanent magnetism, magnetism of induction, and uniform galvanic currents.

It should be stressed that the core element of his innovations consisted in the construction of ‘*neutral language*’ for description and comparison of the consequences from the rival theories and competing theoretical models. Maxwell’s ‘neutral language’ was not Carnap’s and Reichenbach’s ‘observation language’ directly springing out from the ‘*protokolsatze*’ generalizations. Maxwell is aware of the theory-ladenness of the observation data “experimental laws already established, which have generally been expressed in the *language of other hypotheses*” (Maxwell [1861], 1890, p.162). He clearly comprehends that every observation inextricably carries the footprints of the theoretical language that helps to describe it. (‘The daubing of untempered mortar’, as he will call them later in fascinating ‘*Helmholtz*’ paper).

In order to compare and to integrate in a theoretical scheme lacking contradictions all the results of dissent experiments carrying the footprints of multifarious theoretical languages, it is necessary to construct an artificial theoretical language equally distant from the languages of theories under comparison. This stupendous language turned out to be the solid state mechanics (with hydrodynamics as its inherent part). Maxwell’s ultimate aim was to rewrite all the known empirical and theoretical laws of electricity and magnetism applying the neutral language and then to compare them in order to contrive a system without contradictions. The final result of the 1856 magnificent paper was a system of equations lacking the eminent ‘displacement current’. It was not accidental that one of the main drawbacks of the incompressible fluid model consisted in that the latter, apart from some simple cases, was unable to explain interrelations and interactions of electrical and magnetic fields and electric currents, as well as Faraday’s (1845) startling interconnection between optical and electromagnetic phenomena.

Though Maxwellian program’s ultimate goal was to reveal the connection “between electricity at rest and current electricity” absent in the Ampere-Weber electrodynamics. Was it reached already in 1856? – Certainly not. The important connection between the current density $\mathbf{j}$ and the charge density ρ was lacking in Maxwell’s initial 1856 scheme. It was to appear later, after the ‘displacement current’ insertion and finding out its consequence – the continuity equation $\text{div } \mathbf{j} + \frac{\partial \rho}{\partial t} = 0$.

Hence, in 1861 the publication of the second paper (Maxwell 1861) consisting of four parts begins. Its aim was to rederive the results of Weber and Neumann deft theories on the basis of a new mechanical model based on the vortices of incompressible fluid concept.

“My object in this paper is to clear the way for speculation in this direction, by investigating the mechanical results of certain states of tension and motion in a medium, and comparing these with the observed phenomena of magnetism and electricity” (Maxwell [1861], p. 162).

Again and again he has to emphasize that

“the author of this method of representation does not attempt to explain the *origin* of the observed forces by the effects due to these strains in the elastic solid, but makes use of the mathematical *analogies* of the two problems to assist the *imagination* in the study of both” (Maxwell [1861], 1890, p.163; my italics).

The model sprung out of William Thomson’s inquiries. He had revealed that the connection between magnetism and electricity has the same mathematical form as that between certain parts of the phenomena, of which one has a *linear* and the other a *rotatory* character. Note that that Thomson introduced the vortices model in incompressible fluid while investigating Faraday’s experiments on the rotation of the plane of polarized light when transmitted along the lines of magnetic force. Ipso facto, they were the efforts to theoretical reconstruct the Faraday effect that provided the meeting of optics and theory of magnetism. In the second Maxwellian model M_2 magnetic field was represented now by a set of vortices in incompressible fluid with the axes of rotation coinciding with the direction of magnetic field at a point.

“Let us now suppose that the phenomena of magnetism depend on the existence of a tension in the direction of the lines of force, combined with a hydrostatic force [...] The explanation which most readily occurs to my mind is that the excess of pressure in the equatorial direction arises from the centrifugal force of vortices on eddies in the medium having their axes in directions parallel to the lines of force’ (Maxwell 1861, p.165).

But now the important role of neutral language is played not by tube hydrodynamics but by a theory of stresses in the medium where vortices materially represent the vector operator ‘curl’, while the necessary relations among the forces are described by mathematicians with the help of *tensors*. The general type of a tensor describing the general type of stress embraces combination of three principal pressures or tensions, in direction at right angles to each other. The tensor apparatus of solid state mechanics provided contrivance of a ‘dialect’ of new neutral language. It enabled to calculate the force acting on an element of the medium: $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4 + \mathbf{F}_5$.

- (1) The first term $\mathbf{F}_1$ refers to the force acting on magnetic poles;
- (2) the second term $\mathbf{F}_2$ refers to the action on bodies capable of magnetism by induction;
- (3)-(4) the third $\mathbf{F}_3$ and the fourth $\mathbf{F}_4$ terms refer to the force acting on electric currents; (5) the fifth term $\mathbf{F}_5$ refers to the effect of simple pressure that lacks direct electromagnetic analogy. Yet one of the most intricate problems of the vortices model (that puzzled even Daniel Bernoulli who invented it in XVIII century (Whittaker 1910)) was as follows. How can the rotation be transferred from one vortex to another so that “vortices in a medium exist side by side, revolving in the same direction about parallel axis”? – The artful conception that buttressed Maxwell in conceiving this kind of motion was that of the vortices being separated by a layer of particles called the “*idle wheels*”. Is it possible to apply these particles for electricity?

And in the second part of his trailblazing 1861 paper “*The Theory of Molecular Vortices applied to Electric Currents*” Maxwell comes up to the *hardest problem* of his research program: what is “the physical connexion of these vortices with electric currents, while we are still in doubt as to the *nature of electricity*”. It is this point where Maxwell has to trespass the principal *limits* of pure mechanical models and to *insert the alien elements of action at a distance theory*! Or, using the language of the abovementioned lucid methodological model (Nugayev, 1999), we can conclude that Maxwell had to construct the ‘*crossbred models*’ from the languages of *both* cross-theories that combine the properties of completely different theoretical schemes. According to Maxwell’s queer model, an electric current is represented by the transference of the moveable particles interposed between the neighboring vortices. As a result,

“these particles, in our theory, *play the part of electricity*. Their motion of translation constitute an electric current, their rotation serves to transmit the motion of the vortices from one part of the field to another, and the tangential pressures thus called into play constitute electromotive force. The conception of a particle having its motion connected with that of a vortex by perfect rolling contact may appear somewhat *awkward*. *I do not bring it forward as a mode of connexion existing in nature*” (Maxwell [1861], 1890, p. 345; my italics).

On introducing such abstract theoretical objects as ‘electrical particles’ and ‘electric current representing the motion of such particles’ Maxwell had significantly deviated from Faraday’s adroit notions. According to Michael Faraday, the electrical charges should be taken as constructed by the ends of lines of force: they lack an independent substantial existence. Correspondingly, in his *genuine* research program the electric current has to be envisaged not as the motion of real particles but as an ‘energy axis’.

This is the nub of the British field program: the fields are primary, while the particles are only secondary. Maxwell’s blunt eclecticism was followed by H.A. Lorentz’s subtle dualism. Lorentz initiated it in pervasive 1875 paper:

“I shall start with instantaneous action at a distance: thus we will be able to found the theory on the most direct interpretation of observed facts” (quoted from Darrigol, 2001, p.323).

Thereby, it was not a temporary retreat. Even after 1861 Maxwell many times inserted the stiff notions of the Ampère-Weber atomism into his theories (Darrigol 2001).

Yet the results obtained were surely insufficient; the theoretical derivation of Coulomb’s law was apparently lacking. Namely that was done in the third part of profound 1861 paper “*The Theory of Molecular Vortices applied to Statical Electricity*”. Note that the vortices model contained too many ad hoc assumptions. We have now come close to the so-called ‘*Maxwell’s miracle*’.

It turned out that if one (in the course of Fresnel optics and electromagnetism theory meeting) transposes the ether properties from optics to electromagnetism, she can eliminate at least one ad hoc supposition. Indeed,

“it is necessary to suppose, in order to account for the transmission of rotation from the exterior to the interior parts of each cell, that the substance in the cells possesses *elasticity of figure*, similar in kind, though different in degree, to that of observed in solid bodies. The undulatory theory of light requires us to admit this kind of elasticity in the luminiferous medium, in order to account for transverse vibrations. We need then be surprised if the magneto-electric medium possesses the same properties” (Maxwell [1861], p. 13).

This peculiarity has a vital significance for Maxwell’s neutral language:

“If we can now explain the condition of a body with respect to the surrounding medium when it is said to be ‘charged’ with electricity, and account for the forces acting between electrified bodies, we shall have established a connexion between all the principal phenomena of electrical science” (Maxwell [1861], p.13).

So, bold blunt insertion of the molecular vortices theory into the electrostatic realm became possible due to the elasticity of the vortices that enabled the medium to maintain the elasticity waves. As a result,

“the velocity of transverse undulations in our hypothetical medium, calculated from the electromagnetic experiments of M.M. Kohlrausch and W. Weber, agrees so exactly with the velocity of light calculated from the optical experiments of M. Fizeau, that we can scarcely avoid the inference that light consists of the same medium which is the cause of electric and magnetic phenomena” (Maxwell [1861], 1890, p. 22).

The introduction of displacement current was due to Maxwell’s efforts to link the equations relating to electrical current with that of electrostatics. It demanded the Ampère law modification for the sake of a new term introduction; the term had to describe the elasticity of the vortices medium. The driving force due to introduction of displacement current owed its existence to Maxwell’s strenuous efforts to unify all the main empirical laws belonging not only to electricity and magnetism but to optics as well.

As a result Maxwell obtained his immaculate system of equations along with the continuity equation describing the electrical particles that transform the rotations from one vortex to another. But one could not enunciate the final unification of optics and electromagnetism in 1861.

It was possible to profess only commencement of their reconciliation, merely the beginning of ‘grinding’ of rather alien theoretical languages. And at last in 1864 Maxwell had proposed a modified version of his 1861 paper that avoided any special suppositions on the nature of molecular vortices. In his stupendous 1864 paper Maxwell derives his eminent equations from abstract dynamics of Lagrange.

Now he was able to derive the basic wave equation of electromagnetism without any screwy special assumptions about molecular vortices or forces between electrical particles. Although displacement still retained a prominent position in “*A Dynamical Theory of Electromagnetic Field*”, its role differed from that it played in 1861 paper. It was no longer associated with changes in positions of rolling particles.

Rather, Maxwell displayed it simply as the motion of electricity, that is, in terms of a quantity of charge crossing a designated area. Though, despite Maxwell's claim to provide deductions from (three) experimental facts, his account still required the postulation of a displacement current, something that could neither be verified by nor deduced from experiment (Morrison, 2000; Darrigol, 2001).

Finally, I suspect that an important role in 'grinding out' and 'cross-fertilization' of the British (field) and the Continental (action at a distance) research traditions was played by Maxwell's ethnic background. It enabled him to play the role of intermediary in the communication processes. As a British citizen, Maxwell did obtain his scientific degrees at Cambridge. On the other hand, as a patriot of Scotland, he chose the colors of the Scottish national flag for his first color photograph. His critical comments on the British colonial policy in India are well-known too.

REFERENCES

- Audi Robert (ed.) The Cambridge Dictionary of Philosophy. Cambridge, Cambridge University Press, 1999. – 1000 p.
- Boltzmann Ludwig (1895) Ueber Faraday's Kraftlinien, von James Clerk Maxwell. Herausgegeben von. L. Boltzmann. Leipzig : W. Engelmann. – 130p.
- Campbell Lewis, Garnett William (1882) . The Life of James Clerk Maxwell. L., Macmillan. - 342p.
- Cassirer Ernst (1910) Substanzbegriff und Funktionbegriff : Untersuchungen über der Grundfragen der Erkenntskritik. Berlin.
- Cassirer Ernst (1923) Philosophie der symbolischen Formen. Bd. I. Die Sprache. Berlin.
- Chalmers A.F. (1975) Maxwell and the displacement current // Physics Education, January, pp. 46-49.
- Darrigol Olivier (2001) Electrodynamics from Ampere to Einstein. Oxford University Press, 2001. – 515p.
- Feyerabend Paul ([1970], 2010). Against Method. 4th ed., New York, NY: Verso Books.
- Harman P.M. (2001) The Natural Philosophy of J.C. Maxwell. Cambridge University Press. - 228 p
- Hertz H (1884) Ueber die Beziehungen zwischen den Maxwell'schen elektrodynamischen Grundgleichungen und den Grundgleichungen der gegnerischen Elektrodynamik. *Annalen der Physik und Chemie*, 23: 84-103.
- Kant Immanuel [1783] Prolegomena to any future metaphysics that will be able to come forward as science. (Transl. by Gary Hatfield). – In: Immanuel Kant. Theoretical Philosophy after 1781. Cambridge University Press, 2002, pp.29-170.
- Kant Immanuel [1787] The Critique of Pure Reason. The Pennsylvania State University Press, 2010. – 477p.
- Kuhn Thomas S. ([1962], 1970) The Structure of Scientific Revolutions. Second Edition, Enlarged. The University of Chicago Press.

- Kuhn T.S. (1977) Objectivity, Value Judgement and Theory Choice. - In: The Essential Tension. University of Chicago Press, pp. 320 – 339
- Ladyman J. (2008) Structural Realism and the Relationship between the Special Sciences and Physics //Philosophy of Science. - 2008. - 75. – pp. 744-755.
- Lakatos Imre (1978) The Methodology of Scientific Research Programmes. Philosophical Papers, volume 1. Edited by J. Worral & G. Currie, Cambridge University Press, 1978. - 254p.
- Mahon Basil. The Man Who Changed Everything. The Life of James Clerk Maxwell. John Wiley, 2003. – 254p.
- Maxwell J.C. [1856] On Faraday's Lines of Force. The Transactions of the Cambridge Philosophical Society, 1856, vol. X, part 1. Reprinted in "The Scientific Papers of James Clerk Maxwell", 1890, vol. 1, pp. 155-229.
- Maxwell J.C. [1861] On Physical Lines of Force. Philosophical Magazine, 1861, vol. XXI. Reprinted in "The Scientific Papers of James Clerk Maxwell", 1890, vol.1, pp.451-513.
- Maxwell J.C. [1864] A Dynamical Theory of Electromagnetic Field, 1864. Reprinted in "The Scientific Papers of James Clerk Maxwell", 1890, vol. 1, pp.526-597.
- Maxwell J.C. [1873a] A treatise on Electricity and Magnetism. Clarendon Press, 1998. – 532p.
- Maxwell J.C. [1873b] On Action at a Distance. Reprinted in "The Scientific Papers of James Clerk Maxwell", 1890, vol. 1, pp.315-320.
- Maxwell J.C. [1877] Hermann Ludwig Ferdinand Helmholtz. Nature, vol. XV. Reprinted in "The Scientific Papers of James Clerk Maxwell", 1890, vol.2, pp.592-598.
- Mertz John Theodore. A history of European Thought in the Nineteenth Century, 4 vols. Edinburgh: William Blackwood and Sons, 1903-1912. Vols 1 and 2 reprinted as "A History of European Scientific Thought in the Nineteenth Century. NY, Dover, 1964.
- Morrison Margaret. Unifying Scientific Theories: Physical Concepts and Mathematical Structures. Cambridge University Press, 2000. - 270p.
- Niven W.D. A Preface to "The Scientific papers of J.C. Maxwell". Dover publications, NY, 1890, vol.1, pp. XI-XIII.
- Nugayev R. M. A Study of Theory Unification // The British Journal for the Philosophy of Science, 1985a, vol. 36, pp. 159 – 173.
- Nugayev Rinat M. The History of Quantum Theory as a Decisive Argument Favoring Einstein over Lorentz // Philosophy of Science, 1985b, vol. 52, pp. 44 – 63.
- Nugayev Rinat M. Reconstruction of Mature Theory Change: A Theory –Change Model. Peter Lang, Frankfurt am Main. 1999. - 199 p
- Nugayev Rinat M. Copernican Revolution: Unification of Mundane Physics with Mathematics of the Skies. Kazan, Innovative Technologies Publishing House, 2012. – 302 p. (in Russian).
- Nugayev Rinat M. (2018) Einstein's Revolution: A Study in Theory Unification. Bentham Science Publishers, Sharjah, UAE-USA, March 2018
- Nugayev Rinat M. (2020) The Planck-Einstein Breakthrough: Reconciliation of the Pivotal Research Programs of the Classical Physics. Minkowski Institute Press, Montreal, Canada,
- Olson Richard (1975) Scottish Philosophy and British Physics, 1750-1880: A Study in the Foundations of the Victorian Scientific Style. Princeton, NJ, Princeton University Press, 1975. – 349p.
- Planck Max (1922) Die Einheit des physikalischen Weltbildes. Physikalische Rundblicke. - Leipzig.
- Podgoretzky M.I., Smorodinsky Ja. A. (1980) On Axiomatic Structure of Physical Theories. – In : Physical Theory. Moscow: Nauka, pp. 53 – 61 (in Russian).
- Renn, Jürgen (1993) Einstein as a Disciple of Galileo: a Comparative Study of Concept Development in Physics. *Science in Context* 6, 1, pp. 311-341.

Ryckman, Thomas (2005) *The Reign of Relativity. Philosophy in Physics 1915-1925.* - Oxford: Oxford University Press.

Shapiro I.S. (1973) On the History of the Discovery of the Maxwell equations. *Soviet Physics Uspekhi*, vol.15, number 5, pp. 651-659.

Siegel Daniel M. (1991) *Innovation in Maxwell's electromagnetic theory: molecular vortices, displacement current, and light.* Cambridge University Press. - 229 p.

Sigurdson Skoli (2017) *The Nature of Scientific Knowledge. An Interview with Thomas S. Kuhn.* – In: A. Blum, K. Gavroglu, C. Joas, J. Renn (eds.) *Shifting Paradigms. Thomas S. Kuhn and the History of Science.* Editor Open Access. Max Planck Institute for the History of Science.

Stepin, Vyacheslav (2006) *Theoretical Knowledge.* Springer. Volume 326 of Synthese Library.

Tisza L. (1963) The Conceptual Structure of Physics // *Reviews of Modern Physics*, vol. 35, pp.151-185.

Whittaker Edmund Taylor (1910) *A history of the theories of aether and electricity : from the age of Descartes to the close of the nineteenth century.* L., N.Y., Longmans, Green and Co. – 470